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Spline-type Spaces in Gabor Analysis

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Gabor Analysis is concerned with the representation of functions (resp. tempered distributions) f on $\mathbf{R}^d$ as double series, whose terms are obtained from a single building block (the so-called Gabor atom g) by applying time-frequency shifts from some lattice $\Lambda \subset \mathbf{R}^{2d}$. *Good* Gabor expansions make use of a building block g which has both good decay and smoothness properties. As a matter of fact, however, they have to be *redundant* (in particular non-orthogonal), as a consequence of the Balian-Low principle. Equivalently one can describe Gabor analysis as the task of stable recovery of a function f from a sampled (over Λ) STFT (i.e. the *short-time Fourier transform*) $S_g(f)$, for some known g.

Since the short-time Fourier transform is an isometry from the Hilbert space $L^2(\mathbf{R}^d)$ into $L^2(\mathbf{R}^{2d})$ both questions can be treated as problems concerning certain function spaces (containing the functions of the form $S_g(f)$, sharing some smoothness) over the TF-plane.

Although this analogy cannot be used directly

Therefore refined "standard results concerning spline-type spaces" over locally compact Abelian groups have remarkable consequences for Gabor analysis. Among others it will be discussed in which sense the Zak transform finds a natural interpretation in the context of lca. groups. Indeed, this viewpoint allows to clarify under which conditions on a lattices Λ within the abstract time-frequency plane $G \times \hat{G}$ a corresponding *abstract Zak transform* can be found in order to perform Gabor analysis of signals efficiently.