TIME-FREQUENCY ANALYSIS, GABOR MULTIPLIERS AND BANACH GELFAND TRIPLES

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Abstract. Time-frequency analysis (known under many different names, such as Gabor analysis, coherent states, etc.) is a possible way to approach many questions in analysis from a view-point which gives the time-representation of a signal the same relevance as the frequency representation. Even more than classical Fourier analysis the relatively simple algebraic situation that one has over FINITE ABELIAN GROUPS requires a more elaborative (but still very natural) functional analytic approach. Since there are no decent orthonormal bases (unlike wavelet theory) of Gabor type (due to the Balian-Low theorem) on has to accept to work with Banach frames, while on the other hand the question of best approximation of a given operator (in the Hilbert-Schmidt sense) naturally leads to the consideration of Riesz (projection) basis of the corresponding rank-one operators.

It turns out the TF-analysis not only poses a number of delicate questions, but also opens the way to a new Banach Gelfand Triple $(S_0(G), L^2(G), S_0'(G))$ (i.e. a triple, consisting of a Banach spaces of test-functions $S_0(G)$, discovered by the author in 1979, and its dual space, with the Hilbert space $L^2(G)$ in the middle).

It is the purpose of this talk to demonstrate how easy it is to use this Banach Gelfand Triple, how it helps to give a precise mathematical meaning to terms arising in Gabor analysis, and to extend the algebraic facts valid over finite Abelian groups to the setting of general LCA groups.

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