FUNCTION SPACES FOR PSEUDO-DIFFERENTIAL OPERATORS

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Aside from the natural simplicity of the idea of scale space ("constant shape") one of the reasons why *wavelet theory* had an immediate impact was the fact that already in the very first papers the ability of wavelets to characterize the elements of many function spaces considered important at that time (namely $\boldsymbol{L}^p$-spaces, Besov and Triebel-Lizorkin spaces) has been established by Y. Meyer. One can use the continuous wavelet transform or alternatively the wavelet coefficients with respect to a "good" orthogonal wavelet basis. Also the real Hardy space and its dual, the BMO-space can be characterized via wavelet theory, thus establishing the connection to Calderon-Zygmund operators. These are exactly the operators which have a "diagonally concentrated" ' matrix representation with respect to such wavelet bases. Various boundedness results for such operators appear as quite natural under this perspective.d

With *modulation spaces*, introduced by the author already in the early 80's the story went the other way around. First their characterization via Gabor expansions was established, resp. via the short-time Fourier transform, typically using weighted mixed norm conditions. Only long after the basic properties of those spaces had been established it became clear that they are well suited for a description of questions arising in time-frequency analysis, in the theory of slowly time-variant channels (relevant for mobile communication) or for the description of pseudo-differential operators using the Kohn-Nirenberg or Weyl calculus or Sjštrand's class. Compared to wavelet analysis the time-frequency point-of-view allows to tackle similar problems over general LCA (locally compact Abelian) groups, which is not only interesting for the sake of generality, but also because it provides a good setting for the discretization of pseudo-differential operators, providing some insight into the possibility of using finite-dimensional computational methods in order to approximate problems arising in a continuous setting.

In the most simple setting one can use the *Banach Gelfand triple* $(\boldsymbol{S}_0, \boldsymbol{L}^2, \boldsymbol{S}_0')(\mathbb{R}^d)$, consisting of the Segal algebra $\big(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0}\big) = (\boldsymbol{M}^1(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{M}^1})$ (the functions with ambiguity function in $\boldsymbol{L}^1(\mathbb{R}^{2d})$), the Hilbert space $\boldsymbol{L}^2(\mathbb{R}^d)$ and the dual space $\boldsymbol{S}_0'(\mathbb{R}^d)$ of all tempered distributions with bounded short-time Fourier transform.

Coorbit theory provides a variety of other groups where integrable representations provide a corresponding family of Banach spaces and the appropriate atomic decompositions. Shearlets and shearlet spaces are a recent member of this family of coorbit spaces.

It is the author's belief that many more so-called *flexible atomic decompositions* (typically Banach frames for families of Banach spaces) will play an important role for the treatment of possible new classes of pseudo-differential operators and that a better understanding of their properties and of the corresponding atomic (or molecular) decompositions will contribute to progress in this field.

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