# A perfectly invertible and perceptually motivated time-frequency transform for audio representation, analysis and synthesis

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*Time-frequency methods for the applied sciences*

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# Context: Analysis-Synthesis of Sound Signals.

- Audio processing techniques like sound design, audio coding, or speech & music processing require tools to:
  - analyse (represent, extract relevant features...)
  - process
  - re-synthesize sounds
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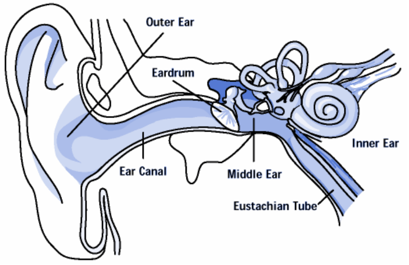
Ideal transform properties:

- Invertibility
- Computational efficiency
- Adaptable redundancy

# The Auditory Resolution.

## 1. Frequency domain: The Auditory Filters.

= Ability to resolve sinusoidal components in complex sounds.



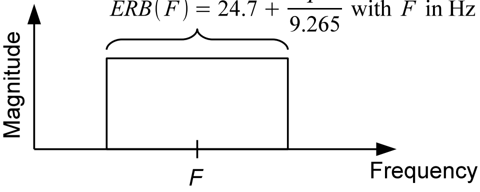

Peripheral filtering ≡ bank of bandpass filters = auditory filters

# The Auditory Resolution.

## 1. Frequency domain: The ERB Scale [Moore & Glasberg, 1983].

**ERB** = **E**_quivalent_ **R**_ectangular_ **B**_andwidth_

$$ERB(F) = 24.7 + \frac{F}{9.265} \text{ with } F \text{ in Hz}$$




$$ERB_{\text{number}}(F) = 9.265 \ln\left(1 + \frac{F}{228.8455}\right)$$

- distribution of filters:
  - $\approx$ linear at low frequencies ($F < 500$ Hz)
  - logarithmic at high frequencies ($F > 2$ kHz)
- $ERB(F) \approx$ constant-Q only at high frequencies

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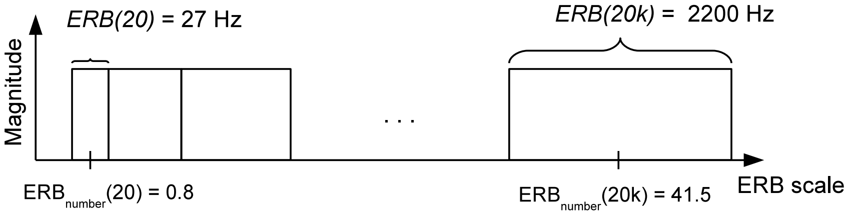


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# The Auditory Resolution.

## 2. Temporal domain.

$=$ Ability to detect rapid changes in sounds over time.

- Frequency partition into filters
  $\rightsquigarrow$ Time windows with frequency-dependent lengths
- **Windows' length = temporal resolution**
- Windows' shape is well approximated by Gaussians with [van Schijndel *et al.*, 1999]:
  - bandwidth $\approx ERB(F)$
  - temporal width $\approx$ 4 periods of $F$, *e.g.*,
    4 ms **@** 1 kHz, 1 ms **@** 4 kHz

# Perceptually Motivated TF Representations.

## State-of-the-Art.

- Auditory models [Plack *et al.*, 2002; Meddis *et al.*, 2012]
  ✓ Useful to gain insights into auditory processing
  X Not invertible, large parameter sets, computationally demanding

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  ✓ Near-perfect or perfect reconstruction
  X Approximate the auditory resolution only at high frequencies, large concentration of filters at low frequencies

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  ✓ Near-perfect or perfect reconstruction
  X Approximate the auditory resolution only at high frequencies, large concentration of filters at low frequencies
- Auditory filterbanks (gammatone, frequency warping) [Smith & Abel, 1999; Hohmann, 2002; Irino & Patterson, 2006]
  ✓ Approximate well the auditory resolution
  X No or only approximate reconstruction

# Goal of the Study.

Achieve a linear TF transform featuring:

- **perceptually motivated** TF resolution
- **perfect reconstruction**
- adaptable resolution and redundancy, *i.e.*,
  - adjustable frequency channels (number of sub-bands)
  - adjustable down-sampling factors

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  - adjustable down-sampling factors

## Proposed approach:

- Use frame theory and the non-stationary Gabor transform (NSGT) [cf. presentation by Peter Balazs] to develop a NSGT matched to the ERB scale
- "ERBlet transform" = *non-uniform auditory filterbank*

# Outline.

1. Underlying concept: The non-stationary Gabor transform
2. ERBlet implementation
3. Simulations
4. Conclusions & perspectives

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# The Non-Stationary Gabor Transform (NSGT).

## Formulation as a Non-Uniform Filterbank [Balazs *et al.*, 2011].

NSG system with resolution evolving across frequency:

$$\mathcal{G}(\mathbf{g}, \mathbf{D}) := (g_{n,k}[l]) = (g_k\left[l - nD_k\right])$$

where

- $l \in \mathbb{Z} =$ time variable
- $n, k \in \mathbb{Z} =$ time and frequency index, resp.
- $\mathbf{g} := (g_k) =$ frequency-dependent filters
- $\mathbf{D} := (D_k) =$ frequency-dependent down-sampling factors

# The NSGT *continued*.

## Frame Theory.

The sequence $(g_{n,k})$ is called a *frame* if the constants $A, B \in \mathbb{R}^{+\star}$ exist that satisfy

$$A\|f\|^2 \leq \sum_{n,k} | \langle f, g_{n,k} \rangle |^2 \leq B\|f\|^2$$

for any signal $f \in \mathbb{R}$.

# The NSGT *continued*.

## Analysis and Synthesis (1/2).

### NSG analysis:

Analysis through the frame operator $\mathbf{S}$ is given by

$$\mathbf{S}f = \sum_{n,k} \langle f, g_{n,k} \rangle \, g_{n,k}.$$

If $\mathbf{S}$ is invertible, then perfect reconstruction is achieved using the *canonical dual frame*

$$\widetilde{\mathcal{G}(\mathbf{g}, \mathbf{D})} = (\tilde{g}_{n,k}) = \mathbf{S}^{-1}(g_{n,k}).$$

### NSG synthesis:

$$f = \mathbf{S}^{-1}\mathbf{S}f = \sum_{n,k} \langle f, g_{n,k} \rangle \, \tilde{g}_{n,k}.$$

# The NSGT *continued*.

## Analysis and Synthesis (2/2).

Conditions for "painless" reconstruction:

- $\hat{g}_k = \mathcal{F}(g_k)$ has a bandpass characteristic
- $\mathrm{supp}(\hat{g}_k) = \mathcal{I}_k$ (in samples)
- $D_k$ satisfies $\lceil \frac{L}{D_k} \rceil \geq \mathcal{I}_k$, $L =$ signal length

It follows that the operator $\hat{\mathbf{S}} := \mathcal{F}\,\mathbf{S}\,\mathcal{F}^{-1}$ is diagonal and easily invertible.

# Outline.


1 Underlying concept: The non-stationary Gabor transform

2 ERBlet implementation
- Analysis & dual windows: ERBlets
- Algorithms

3 Simulations

4 Conclusions & perspectives

# ERBlet Design.

## Analysis Windows.

ERBlet transform $= \mathcal{G}(\mathbf{g}, \mathbf{D})$ with $g_k, k = 0 \ldots K$, defined in the frequency domain by

$$\hat{g}_k[m] = \frac{1}{\sqrt{\Gamma_k}}\, e^{-\pi\left[\frac{m-\nu_k}{\Gamma_k}\right]^2}$$

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To obtain filters equidistantly spaced on the ERB scale:

- Let $F_{\text{min}}$, $F_{\text{max}}$ = min, max analysis frequencies, resp.
- Then $E_0 = ERB_{\text{number}}(F_{\text{min}})$ and $E_K = ERB_{\text{number}}(F_{\text{max}})$
- Distribute $K+1$ filters from $E_0$ to $E_K$ with $V$ filters/ERB
- $\rightsquigarrow E_k = E_0 + k/V$ and $K = V\,(E_K - E_0)$.
- $\nu_k = ERB^{-1}_{\text{number}}(E_k)$
- $\Gamma_k = ERB(\nu_k)$

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Windows truncated so that $\text{supp}(\hat{g}_k) = \mathcal{I}_k = \lceil 4\,\Gamma_k \rceil$.

# ERBlet Design.

## Dual Windows.

“Painless case” condition:

*i.e.*, choose $D_k$ such that the number of time positions

$$N_k = \left\lceil \frac{L}{D_k} \right\rceil \geq \lceil 4\,\Gamma_k \rceil.$$

$\rightsquigarrow$ $\hat{\mathbf{S}}$ is diagonal and easily invertible and $\widetilde{g_{n,k}} = \mathcal{F}^{-1}\hat{\mathbf{S}}^{-1}\widehat{g_{n,k}}$.

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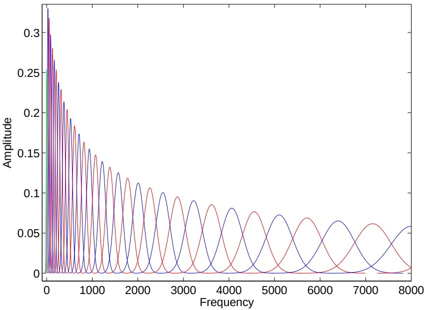
### Otherwise,

if $N_k < \lceil 4\,\Gamma_k \rceil$ then $\hat{\mathbf{S}}$ is not diagonal. We use an iterative method to approximate $\hat{\mathbf{S}}^{-1}$.

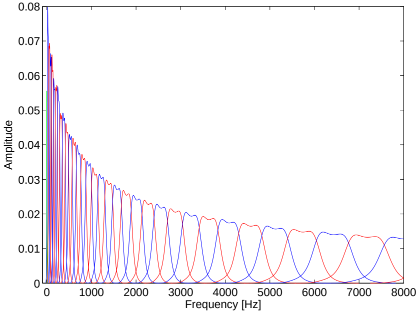
# ERBlet Design.

## Windows Example: Spectral Domain.

Analysis windows



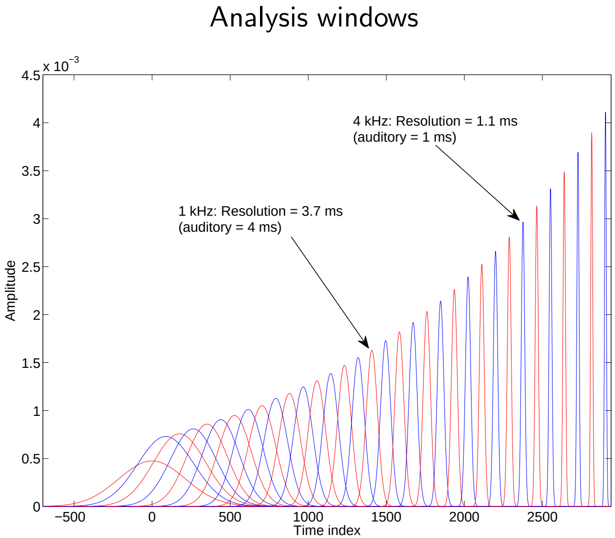

Dual windows




- $F_{\text{min}} = 0$, $F_{\text{max}} = 8$ kHz (Nyquist frequency)
- $V = 1$ filter/ERB ($\equiv$ auditory filterbank)
- $K = 34$ channels

# ERBlet Design.

## Windows Example: Time Domain.



- NSGT with resolution evolving over time available in LTFAT [Søndergaard *et al.*, 2012]: functions `nsdgt.m` and `insdgt.m`
- Applying these algorithms to $\hat{f}$ allows to construct NSGT with resolution evolving over frequency
- ERBlet is determined by 2 parameters: $V$ and $D_k$
  - enable adaptable resolution & redundancy
  - $red = \sum_{k=0}^{K} D_k^{-1}$
  - `erblet.m` and `ierblet.m` soon available in LTFAT

We use a *conjugate gradients* algorithm (CG) to solve the system

$$\hat{\mathbf{S}} f = \sum_{n,k} c_{n,k}\, \widehat{g_{n,k}}.$$

- CG works with Hermitian and positive-definite matrices

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  - we can use `nsdgt.m` followed by `insdgt.m` instead of $\hat{\mathbf{S}}$
- Since $\widehat{g_k}$ decay fast, $\hat{\mathbf{S}}$ is diagonal dominant and

$$\mathbf{P}(\hat{\mathbf{S}})^{-1}_{m,l} = \begin{cases} \left(\sum N_k |\hat{g}_k|^2\right)^{-1}[m], & \text{if } m = l \\ 0, & \text{else} \end{cases}$$

is a good preconditioner [Balazs *et al.*, 2006].

# Outline.


1. Underlying concept: The non-stationary Gabor transform
2. ERBlet implementation
3. Simulations
   - Iterative reconstruction
   - Signal representation
4. Conclusions & perspectives

# Simulations.

## Overview.

### 2 Experiments:

- Exp. 1: Test the convergence of CG for various redundancies
- Exp. 2: Compare the ERBlet to a standard DGT and a linear gammatone filterbank [Hohmann, 2002]

### Setup:

- Audio material: 2 musical excerpts (5–10 sec) in mono format, sampled at 44.1 kHz, 16 bits/sample
- $F_{\text{min}} = 0$, $F_{\text{max}} = 22.05$ kHz

# Simulations.

## Experiment 1: Convergence of CG.

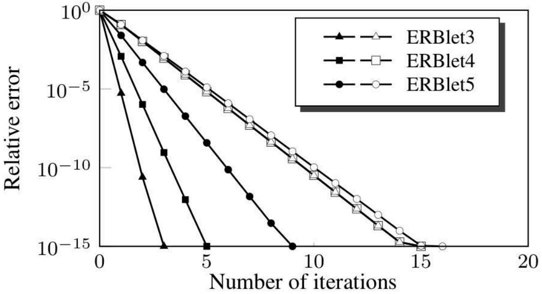



Figure (CG)

| $\mathcal{G}(\mathbf{g}, \mathbf{D})$ | $V$ | $K$ | $N_k$ | $red$ | $B/A$ |
|---|---|---|---|---|---|
| ERBlet3 | 1 | 43 | $\lceil \frac{32\,\Gamma_k}{9} \rceil$ | 3.53 | 1.44 |
| ERBlet4 | 1 | 43 | $\lceil \frac{8\,\Gamma_k}{3} \rceil$ | 2.64 | 1.44 |
| ERBlet5 | 1 | 43 | $\lceil 2\,\Gamma_k \rceil$ | 1.98 | 1.52 |
| ERBlet6 | 1 | 43 | $\lceil \frac{4\,\Gamma_k}{3} \rceil$ | 1.32 | 2.56 |
| ERBlet7 | 1 | 43 | $\lceil \frac{12\,\Gamma_k}{11} \rceil$ | 1.08 | 5.88 |

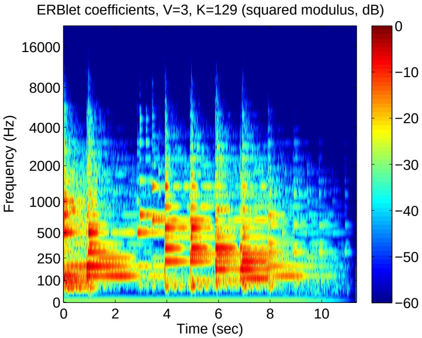
Painless case (reference)

| $V$ | $K$ | $N_k$ | $red$ | $B/A$ |
|---|---|---|---|---|
| 1 | 43 | $\lceil 4\,\Gamma_k \rceil$ | 4.00 | 1.44 |
| 3 | 129 | $\lceil 4\,\Gamma_k \rceil$ | 12.00 | 1.07 |

# Simulations.

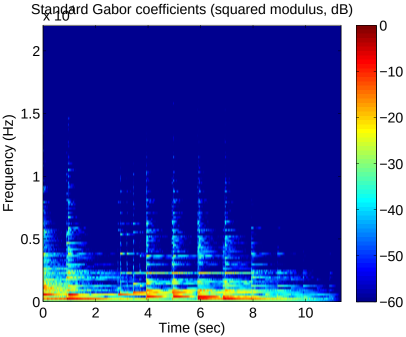
## Experiment 2: ERBlet *vs.* DGT.

ERBlet (dB)




- $B/A = 1.07$ ("painless")
- $red = 12$
- Rel. error $< 10^{-15}$

DGT (dB)

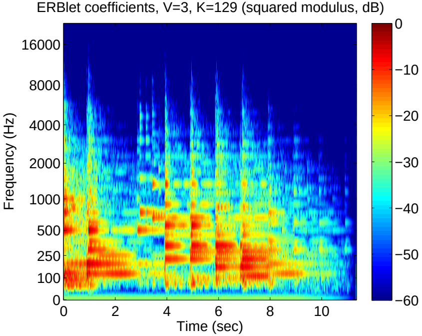



- $B/A = 1.0$
- $red = 11.73$
- Rel. error $< 10^{-15}$

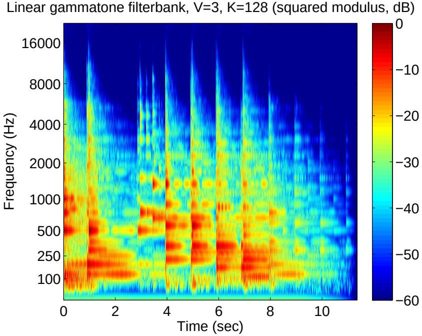
# Simulations.

## Experiment 2: ERBlet *vs.* GFB.

ERBlet (dB)




GFB (dB)




- $B/A = 1.07$ ("painless")
- $red = 12$
- Rel. error $< 10^{-15}$

- $red = 128$
- Rel. error $= 1.3$

# Simulations.

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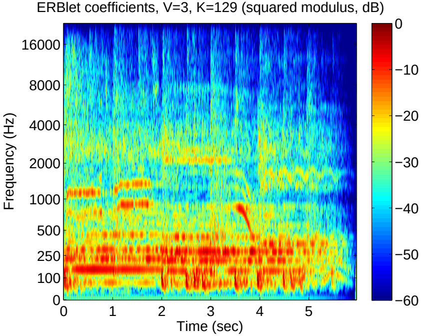


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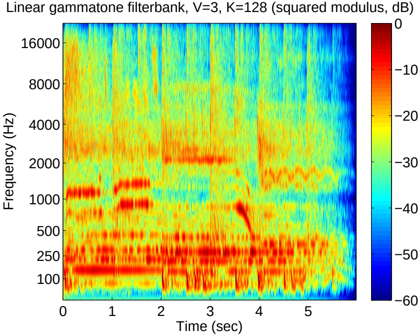


- $B/A = 1.07$ ("painless")
- $red = 12$
- Rel. error $< 10^{-15}$

- $red = 128$
- Rel. error $= 1.4$

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# Conclusions.

- ERBlet = Linear and perfectly invertible TF transform adapted to human auditory perception
- Adaptable resolution and redundancy
  - Perfect reconstruction achieved using iterative method even using 1 filter/ERB and $red = 1.08$
- Compatible with linear gammatone representation
  - Approximates well the auditory TF resolution
- Soon available in the Matlab/Octave toolbox LTFAT
- New analysis/synthesis tool for audio processing

# Perspectives.

- Include basilar membrane compression and compare with nonlinear gammatone filterbanks [Irino & Patterson, 2006]
- Use windows with Gaussian shapes on the ERB scale, *i.e.*, use a warping function to map linear frequency to ERB scale
- Introduce perceptual sparsity in the transform using recent data on auditory TF masking [Balazs *et al.*, 2010; Necciari, 2010]

Thank you for your attention!

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