NuHAG Numerical Harmonic Analysis Group

Sampling theory and applications: developments in the last 20 years and future perspectives

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SampTA13, Bremen, July , 2013

Abstract

At the occasion of the 10th event in a series of conferences, or 18 years after the first SampTA conference (1995 in Riga) it makes sense to look back, and to observe what changed in this period, which dreams have come true (or not) and what the future of the field and hence the **SampTA** conference series can be.

Clearly my views are coming from a rather limited subjective perspective, and I have to ask the audience for excuse for any omissions or mis-interpretations.

Overall I hope to provide a birds-eye view on the subject, reflecting a bit on connections between topics and open issues, and formulate a few desiderata for the field.

Classical Motivation

The classical motivation for the irregular sampling problem comes of course from variations on Shannon's Sampling Theorem, which mathematically speaking is a consequence of Poisson's formula. In short it provides a concrete answer to the problem of complete recovery of a band-limited function from regular samples, by providing a series expansions of those band-limited functions, using translates of a given function (e.g. the classical SINC-function), if only the so-called Nyquist criterion is satisfied.
Still one has to cope with different kind of errors: Can one control the error if the function is not perfectly band-limited (aliasing error) or if the sampling location is not exactly known or used (jitter error). Moreover, in practice only finitely many data are available (truncation error), requiring function space methods.

Irregular Sampling and Scattered Data Approximation

Although the problem of exactly interpolating finite data by polynomials of a corresponding degree has a unique answer (Lagrangian interpolation) it often does NOT provide the expected answer, which may be more like fitting the data in the sense of polynomial regression.
Therefore the irregular sampling problem as understood in "our community" is often better presented as scattered data approximation. The task is to find a smooth function which optimally fits a collection of noisy point values.
Sampling theory has become a field within analysis which interacts with a wide range of topics and methods. In many cases it is a testing place of ideas that apply potentially in a much broader context, while on the other hand abstract ideas are relevant for the practical solutions of irregular sampling problems.

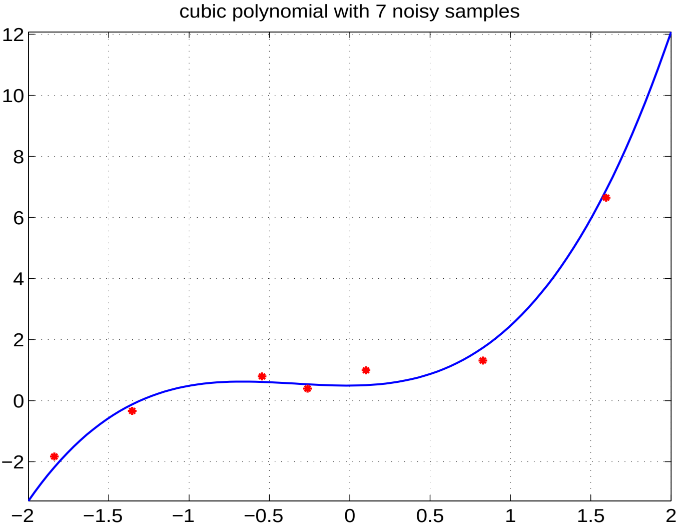
What are the Questions?

- Under which conditions on the function f and the sampling family $X = (x_i)_{i \in I}$ can one guarantee reconstruction of f, either "perfectly" or with some error (which however should be controlled); Very often the model assumptions (band-limited, spline-type, etc.) come into play here;
- Hopefully the recovery can be done in a linear way, possibly with efficient iterative algorithms;
- Even if such algorithms are described mathematically the questions from the community will be: what is a good implementation of such an algorithm, what kind of memory requirements does one have, etc.

What are the Questions II?

Of course also the question of what our main questions can be stretched. In many cases it is not point measurements (i.e. samples), but rather localized information about the underlying (smooth) function.
Hence an important variant of the sampling problem is the question of recovery of a function in a given space from local average. Depending on the application these averages are of constant width (easy case) or vary from place to place (different devices, or potentially different reliability).
I will not go deeper into this interesting subject, but mention two aspects: First of all it indicates that we are really doing functional analysis here. On the other hand many of the existing results solely rely on the diameter of the averaging window, and thus cover jitter error as special case, while concrete measurement devices may not be so bad!

What are the Tools? A Mini-Problem first:



What are the Tools? A Mini-Problem first:

It may be useful to recall the situation in the setting of linear algebra. Here we have the advantage of a being able to understand everything in terms of matrices, and we can do all the computations using MATLAB or OCTAVE, compute eigenvectors of corresponding (frame) matrices and check their condition numbers.

Finding the solution of the MNLSQ-problem to the problem of finding a cubic curve, given 7 noise samples of such a curve may be a good setting to teach students what irregular sampling resp. regression is all about. But it is clear that here the pseudo-inverse of the corresponding matrix may be a good answer. Of course, later on, we will interpret this as making use of the canonical dual frame.

What are the Tools I?

The first question is about the model for the space of functions (in a general sense) to which the sampled signal belongs. There are in fact many choices:

- the classical setting of band-limited functions;
- the setting of spline-type space (PSI or wavelet spaces);
- or just RKH (reproducing kernel Hilbert spaces) over some domain, among them Hilbert spaces of analytic functions;
- often a *transformation* which allows to identify the given Hilbert spaces with a RKH over another domain: see STFT or CWT, transforming $\mathcal{H} = \mathbf{L}^2(\mathbb{R}^d)$ into a space of continuous functions in $\mathbf{L}^2(G)$, for suitable groups G ($ax + b$ resp. Heisenberg group).

What are the Tools II

Within a given class of functions we often find a natural description of their decay or summability properties in terms of (e.g. weighted $\mathbf{L}^p$-norms). For so-called *well-spread* point sets one then typically finds that corresponding discrete norms (weighted ℓ^p sequence space norms), applied to the family of sampling values $(f(x_i))_{i \in I}$ provide equivalent norms.
Of course there is a large variety of possible function spaces, but I would recommend not to invest to much effort in increasing the number of parameters, but rather make generic assumptions on the function spaces used (e.g. solidity, translation invariance, etc.).

At this place I have to strongly urge everybody to look for solutions for families of function spaces and not just for arbitrary (but fixed) individual function spaces!

Doing Generalizations properly!

Over the years we have learned to extend results first obtained in the context of some Hilbert space to more general function spaces. But how should such extensions be done? Replace $\mathbf{L}^2(\mathbb{R})$ by $\mathbf{L}^2(\mathbb{R}^d)$, by $\mathbf{L}^2(G)$, or even some weighted $\mathbf{L}^p$-space (G LCA).
Let us look at some examples:
The first one is just the $\mathbf{L}^p$-variant of Shannon's classical theorem:

Theorem

Given $p \in (1, \infty)$ and $\alpha \leq 1$ there exists some constant $C_p \geq 1$ (depending on p and α) such that one has

$$C_p^{-1} \|f\|_p \leq \left(\sum_{n \in \mathbb{Z}} |f(\alpha n)|^p \right)^{1/p} \leq C_p \|f\|_p.$$

BUT DOES SUCH A THEOREM HELP?

Doing Generalizations properly II

Think of various scenarios:

- You obtain some data, and would like to recover f from the data. *Will the user provide the INFORMATION to you:* "I took the samples from a function in $L^{5/4}(\mathbb{R})$?
- Even if we assume this oracle to be available, what is the actual help from this statement? Will the constant $C_{5/4}$ be large (because $p = 5/4$ is not so far away from $p = 1$)?
- As a matter of fact one can show that $C_p \to \infty$ as $p \to 1$?
- Even if this situation would be answered in a satisfactory way it would just allow as to state that the set of samples is a closed subspace of $\ell^p(\mathbb{Z})$. But what happens if we get slightly perturbed data not in the range? Can we then just project them to the range of the sampling operator? *Unfortunately there are plenty of non-complemented closed subspaces for $p \neq 2$.*

Doing Generalizations properly III

In this sense a theorem of the following type is much more useful (and actually used in applications):

Theorem

For any $\alpha < 1$ *there exists some constant* $C_\alpha \geq 1$ *such that*

$$C_\alpha^{-1}\|f\|_p \leq \left(\sum_{n\in\mathbb{Z}} |f(\alpha n)|^p\right)^{1/p} \leq C_\alpha \|f\|_p,$$

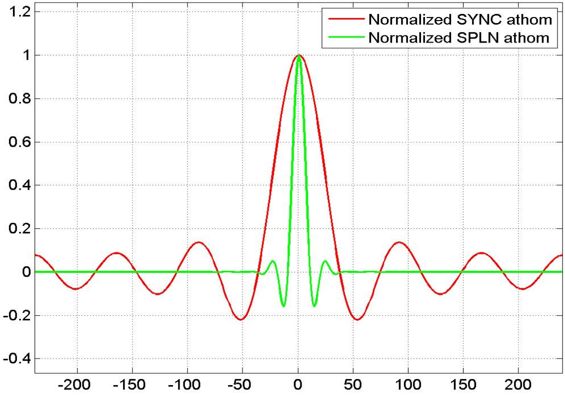
and furthermore, for atoms g chosen properly one has an unconditionally convergent series representation

$$f = \sum_{n\in\mathbb{Z}} f(\alpha n) T_{\alpha n} g, \quad \text{for} \quad f \in \mathbf{L}^p(\mathbb{R}),$$

assuming only that $\mathrm{spec}(f) \subseteq [-1/2, 1/2]$.

Illustration of SINC versus better atoms:

The alternative kernel is of course much better concentrated, but it has a (much) larger spectrum in the particular case:



Doing Generalizations properly IV

Let us shortly analyze the situation and learn from it:

1. In the first version, the reconstruction standing behind is an extension of the *ideal recovery system*. The collection of shifted SINC-function is an ONB for the space of band-limited $\mathbf{L}^2$-functions with $\mathrm{spec}(f) \subseteq [-1/2, 1/2]$. So what kind of problem do we get: it the fact that $SINC \notin \mathbf{L}^1(\mathbb{R}^d)$!! (which causes the problem and even prohibits a proper recovery for $p = 1$ because finite partial sums are not in $\mathbf{L}^1(\mathbb{R})$).
2. In the second case on makes use of the slight redundancy and replaces $\mathcal{F}(SINC)$, i.e. the box-function, by a smooth version of a box-function $\hat{g}$, thus buying some decay, hence integrability of g, and getting the second theorem.

Doing Generalizations properly V

The wish to have a uniform statement for families of function spaces (here the family of $\mathbf{L}^p$-spaces for $p \in [1, \infty]$ requires to pay a price (allow a bit or redundancy), **but** if properly handled this redundancy implies a variety of beneficial properties):

1. Better robustness with respect jitter- or aliasing errors;
2. validity of the recovery not only for $\mathbf{L}^p$-spaces, but families of (moderately) weighted space $\mathbf{L}^p{}_m(\mathbb{R}^d)$, as long as

$$m(x+y) \leq (1+|x|)^s \cdot m(y) \quad \forall x, y \in \mathbb{R}^d.$$

3. as well as locality of the representation: *Having only local data one is still able to recover the underlying function f using the relevant partial sums; up to boundary effects the reconstruction error is small for the region covered by the available sampling points!*

Doing Generalizations properly VI

Summarizing we argue, that a variation of the classical approach suggests to look out for linear recovery methods which apply universally to families of functions of a particular type (e.g. band-limited functions) an a well-chosen family of function spaces (such as polynomially weighted $\mathbf{L}^p$-spaces over $\mathbb{R}^d$, up to some order s_0).
Such methods (whether they are obtained by direct methods, such as local regression etc. and/or iteratively) should be (and often are automatically by their very nature) local and robust with respect to the natural errors, such as noisy data, jitter or aliasing error (which can be viewed as mild deviation from the signal model).

The settings

So far we have discussed the case of band-limited functions, of you want over LCA (locally compact Abelian) groups, but most of the constructive (often iterative) methods are of the type: Assuming some density (typically a reasonable fraction of the Nyquist rate) one is able to guarantee (uniformly over the said family of function spaces) uniform rates of convergence (e.g. geometrical rate of convergence).
The key argument in many of the existing proofs is based on the consideration of the smoothness of the underlying function and the fact, that it cannot vary too strongly over the Voronoi-regions of each point, implying that the Voronoi step-function is not far from the original function if the point density is high enough!
In many case the oscillation of f is estimated using some form of Bernstein's theorem.

The settings II

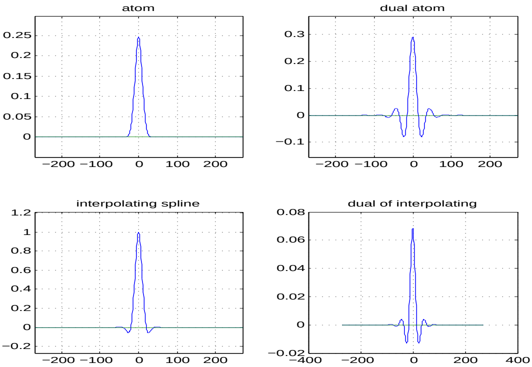
The early papers on iterative methods (first generation methods) consist in building any form of (quasi)- interpolation to the original function of the form $\sum_{i \in I} f(x_i)\psi_i(x)$ which is then "projected" back to the space of band-limited functions.
Such an approach however requires only (as reported at the Loen conference) three things:

- Control of the oscillation and the size of the sampling values;
- The solidity of the function spaces, because only pointwise error estimates can be obtained (providing norm estimates);
- Some (kind of) projection operators from the ambient $\mathbf{L}^p_w$-space into the subspace under consideration (e.g. band-limited functions).

The settings III: Spline-Type Spaces

Technically speaking Wiener amalgam spaces are well suited to handle such problems and describe them properly, specifically the spaces $\mathbf{W}(\mathbf{C}_0, \ell^p_m)(\mathbb{R}^d)$.
An important family where such principles apply is the family of what I call spline-type spaces, called (principal) shift-invariant or wavelet spaces elsewhere. The prototype is of course the family of cubic spline functions, which thanks to the good properties of B-splines have the nice property that they are in $\mathbf{L}^p_w$ if and only if their coefficients (with respect to the basis of shifted cubic B-splines) is in the corresponding sequence space ℓ^p_w.
Properties of the dual generator, i.e. the function in the spline-type space whose translates are a biorthogonal basis to the generating system (they are also in $\mathbf{W}(\mathbf{C}_0, \ell^1_w)$) play a role.

The settings III: Spline-Type Spaces



The settings IV

Algorithmically the change from band-limited functions to spline-type functions was quite easy (and that is how it started!), for the first generation algorithm. One just has to replace the filtering step by a projection onto the space under consideration, making use of the fact that this is a bounded operator (uniform with respect to the family under consideration).

Of course such considerations can be carried out over LCA groups and with rather general kernels. Let us just mention the reproducing kernel of Sobolev space (if w is the weight on the FT side it is $\varphi = \mathcal{F}^{-1}(1/w^2)$). It follows that minimal norm interpolation of equidistant ℓ^2-data is by means of the spline-type space generated by φ.

The settings V

These considerations bring us already close to two important further topics/connections

The first one is the connection to RKH, i.e. the theory of reproducing Hilbert spaces. In some sense the irregular sampling problem for spaces of band-limited functions is equivalent to the stable representation of signals as series of shifted SINC-functions, with ℓ^2-coefficients.
Here the notion of "stable generators for a Hilbert space" comes into game. We all know, that this is just the other side of the medal of frame theory, in fact the slightly less well known.

So let us shortly discuss some aspects of the theory of frames (and Riesz basic sequences).

Frames and Riesz Bases I

Almost every mathematics student is starting her/his career with some course in analysis and **basic concepts of linear algebra**. From the concept of a vector space it is just two steps away to that of a linear independent resp. generating set of vectors, and in the ideal case on has both properties, i.e. one has a basis. Every finite-dimensional vector-space has such a basis and consequently all linear mappings between such spaces can be described by matrix. The composition law of linear mappings enforces a specific for of *matrix multiplication* resp. matrix inversion, and e.g. the determinant is the standard way to check the invertibility of a square matrix. We also learn that orthonormal basis (resp. unitary matrices) are the best thing we can get, because one obtains U^{-1} simply by forming $U' = conj(U^t)$.

Frames and Riesz Bases II

Although we have learned in our *functional analysis courses* that the concept of orthonormal bases is still available for abstract Hilbert spaces $\mathcal{H}$ (and this is OK, they behave as in the linear algebra situation), we are *falsely* indoctrinated that one should

- linear independence carries over by just looking at arbitrary finite subsets;
- the concept of a generating set has to be replaced by the concept of a *total family* M , which means that $\mathcal{H}$ concides with the closed linear span of such a family M.

Frames and Riesz Bases III

But can we then transfer basic observations from linear algebra into this setting.
Maybe you may think of the critical Gabor family of shifted Gaussians along the Neumann lattice $\mathbb{Z}^2$, which was suggested by Denis Gabor. It IS linear independent (in the classical sense), and it is total, but is far from being a basis of any kind. One can even remove one of those vectors (but not two of them!) and still have a total family. But on the other hand there is no stable way of representing elements from $\mathcal{H} = \mathbf{L}^2(\mathbb{R})$ using ℓ^2-coefficients!

Frames and Riesz Bases IV

We all know that the correct replacement of these two concepts are injective mappings between the Hilbert space and some $\ell^2(I)$ spaces. If we consider the **stable** variant of linear independence, then we assume that the mapping from $\ell^2(I)$ into $\mathcal{H}$ which assigns to $c = (c_i)_{i \in I}$ the vector $\sum_{i \in I} c_i \varphi_i$ (forming infinite linear combinations, which is the natural analogue of matrix multiplication) defines an imbedding of $\ell^2(I)$ into $\mathcal{H}$.

This can be characterized by a well-known pair of inequalities, and we are talking about a Riesz basic family.

Quite analogously a family $(g_i)_{i \in I}$ is a frame if the mapping $f \mapsto (\langle f, g_i \rangle)_{i \in I}$ defines an embedding from $\mathcal{H}$ into $\ell^2(I)$.

Frames and Riesz Bases V

Although not with this (by now well established) terminology these two fundamental questions had found already considerable attention in the community of colleagues interested in Hilbert spaces resp. Banach spaces of analytic functions, on the plane (e.g. Fock-spaces) or on the (open) unit disk.
There the corresponding terminology describes results concerning sets of sampling and sets of interpolation. In the first case stable recovery from the samples is guaranteed, using series expansions with ℓ^2-coefficients (resp. frames of reproducing kernels). In the second case the Riesz basis situation is described by the fact, that for arbitrary ℓ^2-data a function f can be provided which takes those given values at the given positions.

Frames and Riesz Bases VI

Many of those transformations arise form irreducible and (square) integrable (projective) representations of certain (non-Abelian) groups:

- coherent states (resp. time-frequency analysis), using the Schrödinger representation of the reduced Heisenberg group;
- poly-analytic functions (using Hermite atoms);
- $ax + b$ group leading to the continuous wavelet transform, defined on the upper half-plane (identified with this group);
- shearlet transform, allowing to describe shearlet frames via sets of stable sampling in that domain;
- Blaschke group, with a corresponding voice-transform;

Frames and Riesz Bases VII

It is clear that this is the point where strong relationships between harmonic and complex analysis play an important role, which is still gaining momentum.

Typically hard analysis methods and function space methods allow to prove *qualitative results* in much greater generality (meaning transform need not be analytic, but still interesting sampling results can be shown), but under stronger assumptions concerning the density, while complex analysis methods allow to prove optimal results in terms of density.

The classic here is the sampling of the STFT with Gaussian windows. Complex analysis methods provides the optimal answer for regular lattices of the STFT: any lattice of the form $a\mathbb{Z} \times b\mathbb{Z}$ with $ab < 1$ is a set of sampling.

But real analysis methods allow to verify that for $a = b < 0.99$ the situation is OK!

Frames and Riesz Bases VIII

There is an interesting connection between sampling the STFT with respect to higher order Hermite functions (the Gauss function is considered as the Hermite function with index zero!) and the theory of polyanalytic functions which has found great attention very recently, leading to yet another bridge between sampling theory, coorbit theory and complex analysis methods.

Another large branch of science very close to sampling theory which cannot be presented here is learning theory. We had invited talks in that direction at earlier SampTA conferences, and e.g. generalized sampling, already seen at this conference.

Also on non-Abelian groups or manifolds it is possible to define band-limited functions via the *spectral decomposition of the Laplacian operator.*

Density of Sampling Sets I

One of the universal questions arising in sampling theory is the question of the existence of something like a Nyquist rate. This means (e.g. in the band-limited case) that one hope for a Riesz basis, or ideally an ONB for the Hilbert space under consideration, such as $T_n(SINC), n \in \mathbb{Z}$.

As already mentioned this setting is not extending to the case of band-limited functions in $\mathbf{L}^1(\mathbb{R}^d)$, so somehow if one is interested in uniform claims for the families of band-limited function in

$$\{\mathbf{L}^p(\mathbb{R}^d) \,|\, p \in [1, \infty]\}$$

the answer is quite different. We have sets of sampling (resp. frames) for regular sampling below the Nyquist rate and sets of interpolation above the Nyquist rate.

Density of Sampling Sets II

This will of course remind many of you of the well-known Balian-Low phenomenon: there is no well Gabor-like Riesz basis for $\mathbf{L}^2(\mathbb{R})$ generated from an atom/window which is well-localized in the TF-sense.

We also remind the audience that Yves Meyer was trying to show that a similar claim is also valid in the case of the CWT, and the failure of his assumption was due to his own construction of an orthonormal wavelet system

Recall that the search of coherent frames (such as Gabor or wavelet frames) is equivalent to the search for sets of sampling for the corresponding continuous transform, with the atoms being called windows in the transform setting. Correspondingly we are looking for sets of interpolation when we search for coherent Riesz basic sequences.

Density of Sampling Sets III

While Balian-Low is seen as a deficiency it can also be seen as a natural consequence of "good properties" of Gabor systems with respect to dilations: It is known for a while that any regular Gabor system (i.e. a system $(\pi(\lambda)g)_{\lambda\in\Lambda}$, with a lattice $\Lambda \lhd \mathbb{R}^d \times \widehat{\mathbb{R}}^d$) generating a Gabor frame with a "decent window" (with $g \in \mathbf{M}^1(\mathbb{R}^d)$, e.g. $g \in \boldsymbol{\mathcal{S}}(\mathbb{R}^d)$) allows some small perturbation by some matrix, as long as that matrix is close to the identity matrix. In particular, it is always possible to apply a mild stretching, which in fact changes the density of the family. More recently the same has been proved for non-regular Gabor families.
On the other hand it is known that for point sets with subcritical density one cannot have a Gabor frame. Combining these observations one reaches the fact that a Gabor frame must have some redundancy, while a Riesz basis must have at least critical density.

Density of Sampling Sets IV

Overall there are, depending on the setting and scenario different types of density descriptions for sets of sampling, often directly or indirectly based on the ideas laid in the early work of Beurling and Landau.

Moreover it may not be surprising that similar are not available in the case of the affine group, and even the proper concept of density is not completely clear. In any case no stability with respect to density changing transformations in conjunction with the existence of a kind of Nyquist density (whatever it may be) can be valid, otherwise we would not have (plenty of) nice orthonormal wavelet bases.

Density of Sampling Sets V

The TF-context also allowed to formulate quite strong results in the following spirit: If we have a Gabor frame of some redundancy, then it is always possible to remove a certain percentage of points as long as there is still some redundancy left over. Although in this way the condition number grows with decreasing redundancy it is still possible to reach a given level of (low) redundancy.

This raises of course the question: In which other context can one play such games. Is it something very specifically to the Weyl-Heisenberg setting. Most likely it applies to many more settings, but perhaps not to all of the cases where sampling has its natural place!? There are more open questions than answers if we ask the question in this generality.

A Conjecture based on Sampling Sets

We have many cases where any set of sampling is well spread, in the sense of having an upper limit on the local density. Since it is also often not too difficult (using e.g. the concentration of a family of reproducing kernel elements) to show that sufficiently separated define sets of interpolation, it was not so courageous to hope that CONJECTURE: **Every frame in a Hilbert space is a finite union of Riesz basic sequences.**

While only few weeks ago I would have said that one can gain high reputation by answering this question (negatively or positively), one can now say that the Kadison-Singer conjecture and hence the above CONJECTURE has been verified affirmatively:

Interlacing Families II: Mixed Characteristic Polynomials and the Kadison-Singer Problem, by Adam Marcus, Daniel A. Spielman, Nikhil Srivastava (Arxiv, June 2013).

Localization I

Another topic within sampling theory where I see substantial progress in the last decade is the so-called localization theory. Similar to the scenario described in the context of Shannon's theorem with the chance for better localized building blocks one can formulate the following question (in the most abstract setting):

PROBLEM: Given a sufficiently dense subset from a RKH producing a frame of some redundancy, hence many possible dual frame families. What is the most concentrated dual family, resp. what can one say about the concentration of the *canonical dual* frame (with respect to the centers).
Especially in the context of Gabor analysis the uniform concentration of dual Gabor families (also in the irregular case), related also to the so-called HAP (homog. approximation) property, has been carefully investigated.

Localization II

This newly established localization theory provides concepts of localization of a frame with respect to a fixed orthonormal basis (could be a localized Fourier basis for TF-applications), or intrinsic or mutual localization of frames.

Technically it amounts to the question of spectral invariance in suitably chosen Banach algebras of operators or infinite matrices, describing good off-diagonal decay in the sense of the underlying group.

Clearly the positive answers to such questions (often viewed as generalizations of Wiener's inversion theorem) provide insight into the locality of reconstruction, i.e. the guaranteed quality of reconstruction in certain regions (up to some boundary effects) if only local data are available.

FURTHER TOPICS I

Areas where we have seen substantial progress in the two last decades are (without claiming completeness of the list):

- random sampling is now much better understood, but it may still worthwhile to view irregular sampling algorithms from a more probabilistic point of view; in fact, most of the analysis is still going into worst case analysis;
- Sampling in the context of finite rate of innovation has found some attention;
- The principle of consistent sampling has been developed and is by now well established (if perfect recovery is impossible);
- The effect of quantization, in particular the possibility of good recovery of a signal from densely sampled but coarsely quantified signals has bee studied (see Sigma-Delta Quantization);

FURTHER TOPICS II

Of course one of the really blooming (not to say hot) topics which may be considered an offspring of sampling theory is

- compressive sensing resp. compressive sampling, with the astonishing idea that may not be necessary to fully recover a signal in order to extract certain features from it;
  In combination with convex optimization algorithms there is by now a wide range of applications, we see books coming up in the field and lots of workshops and conferences;
- also highly relevant for various application areas is the phase retrieval problem: when and how can we recover a signal from the absolute values of the samples only ;
- new mathematical foundations of superresolution, there will be a full session at SampTA13;

FURTHER TOPICS III

The above mentioned topics are already well established, but we see other questions coming up, and at least I find them quite interesting and see them as challenges for the future:

- distributed processing of sampling (local regression, independent fusion centers, fusion frames, etc.); algorithmically this is of course also related to the question of parallel algorithms for scattered data approximation, e.g. in the context of higher dimensions (GPU implementations?);
- real time processing does not only require fast algorithms and good hardware, but in the analysis of such algorithm one has to observe that parts of the data are not yet available when the processing starts, and a continuous flow of output may have to be created!

OUTLOOK I

Even with such good progress and many interesting new topics developing into well-founded small theories with nice applications I would like to express some wishes/challenges for the future:

1. We have strong theory and in some cases even better performance of algorithms. *But have we systematically verified using numerical methods that our theoretical estimates are close to optimal?* Maybe in some cases one can even justify the rates of convergence (and even constants) are (at least qualitatively) optimal!
2. When we talk to applied people we can tell them a lot about a huge variety of ideas and methods, but can we provide them with a reliable consumer report, indicating which method is the best in which situation? Maybe "grand challenges" put together in order to identify best practice could be helpful in this respect;

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