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A Banach algebra A has the (weak) factorization property if for all $f \in A$ there exist g,h

$$(g_1,\dots,g_{n(f)},h_1,\dots,h_{n(f)}), \quad \text{such that } f=gh \ (f=\sum_{i=1}^{n(f)} g_i h_i).$$

According to Cohen's theorem the existence of bounded approximate units implies factorization. Examples of Paschke and Leinert have shown that the converse is not true without any additional condition. It is shown, that most of the natural group algebras of a locally compact abelian group allow to prove a converse to Cohen's Factorization Theorem.

<u>Theorem 1.</u> B Segalalgebra, $B \subseteq L^p(G)$ for some $p > 1 \Rightarrow$ B has not weak factorization.

<u>Theorem 2.</u> B Segalalgebra, character-invariant i.e. $\chi f \in B$ for all $f \in B$, $\chi \in \hat{G} \Rightarrow$ B has not weak factorization.

The proof of Theorem 2 requires some results on the multipliers from L^1 to a Segalalgebra B. The general result reads as follows:

<u>Theorem 3.</u> Let B be a dense Banachalgebra $B \subset L^1(G)$, $\| \; \| \geq \| \; \|_1$ satisfying

1) $L_y B \subseteq B$ for all $y \in G$, $\lim_{y\to e} \|L_y f - f\| = 0$ for all $f \in B$;

2) $M_\chi B \subseteq B$ for all $\chi \in \hat{G}$; $\lim_{\chi\to\hat{e}} \|M_\chi f - f\| = 0$ for all $f \in B$; $\sum_{n=1}^{\infty} \frac{\log\|M_\chi n\|}{n^2} < \infty$ for all $\nu \in \hat{G}$.

3) $\|f^* * f\| \leq K\|f\|^2$ for all $f \in B$;

Then the following conditions are equivalent

I) B has (weak) factorization

II) B has bounded appr. units

III) $B \supset L^1_w(G)$ for some weight function w.

(One may suppose that w is a continuous function on G satisfying $w(x) \geq 1$ and $w(x+y) \leq w(x)w(y)$ for all $x,y \in G$).