General Setting of my (Opening) Talk

Since this is a mathematical conference, in fact a conference for experts in frame theory, with many of my friends and former students present, who have contributed greatly to this (fashionable and important) field, I would like to take an outside viewpoint.

I would like to share with you some thoughts, which at the end will help to grasp better the potential of frame theory, aside from possible extension and generalizations which are flooding the journals.

The goal is to show how and why **Banach frames** for **families of Banach spaces** are an important practical concept. But in order to avoid the handling of two many parameters (including weighted, mixed-norm spaces, as we used then in the theory of irregular sampling and in coorbit theory) I reduce my consideration to the setting of a very specific Banach Gelfand Triple, based on the Segal algebra $\left(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0}\right)$.

NuHAG Numerical Harmonic Analysis Group

Banach Gelfand Triples and Rigged Hilbert Spaces

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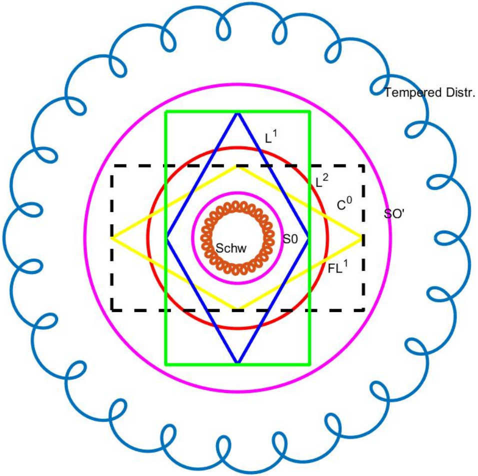
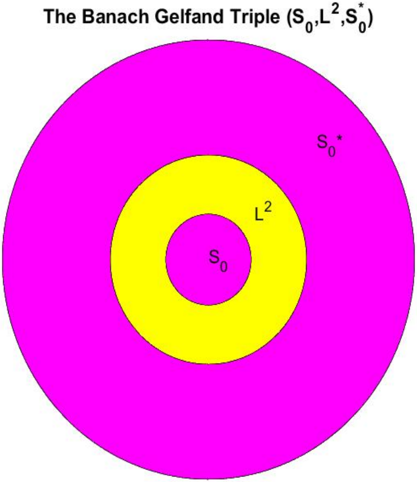
Talk held at FWAMA2019
Palermo, Sept. 16th, 2019

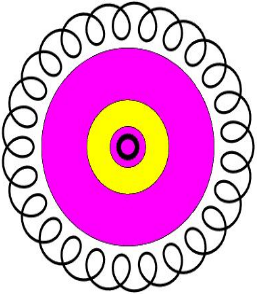
A Zoo of Banach Spaces for Fourier Analysis

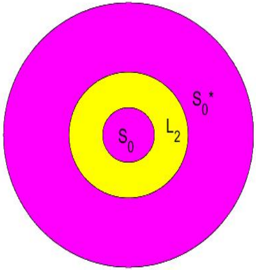


The S0-Banach Gelfand Triple



The S0 versus Schwartz Gelfand Triple





Our number system

What is π?

Can we say that $\pi = 3.1416$, as MATLAB tells us? Or $\pi = 3.141592653589793$ if we put `format long`!

Note, both are rational numbers, which have an inverse, which is (again, according to MATLAB)

$$0.318309141838554 \quad \text{resp.} 0.318309886183791$$

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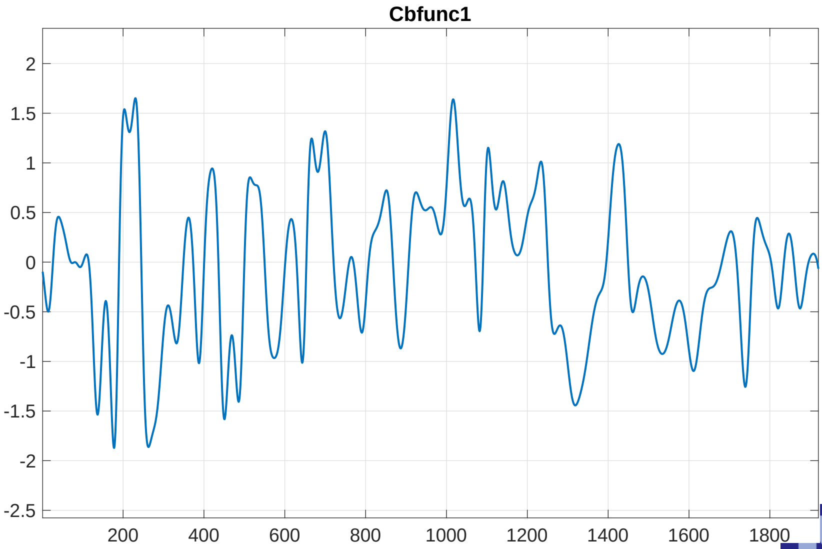
Infinite Decimals

The usual, convenient way to define the real numbers is to define them as “infinite decimals” (resp. as the natural limit of arbitrary long decimal expressions).
Keeping in mind that our decimal system is not “good-given” (we might use other number systems, or simply other ways to describe suitable rational numbers) one easily comes to the conclusion, that we have to consider equivalence classes of Cauchy-sequences of rational numbers. For them one has to introduce all the relevant operations (e.g. multiplication and taking the inverse) and to verify that they form a *field*, called $\mathbb{R}$!
So `real numbers` are not as real as one might think
(in the sense of concrete objects).
Still, these symbols (like $1/\pi$) are not just useless conventions!

A thought experiment

`A thought experiment (German: Gedankenexperiment) considers a hypothesis, theory, or principle for the purpose of thinking through its consequences.`

Assume you do not know what Lebesgue spaces are, not topological vector spaces, test functions or tempered distributions.
What are the objects, which have a (bounded) spectrogram?
Certainly we can expect that a piece of music, or the signal coming from an ECG, could be corresponding objects, which can be recorded and subsequently analyzed. They are NOT almost everywhere defined functions in $\boldsymbol{L}^2(\mathbb{R})$!
First of all let us agree for a moment that we take the spectrogram using smooth plateau-functions, adding up to constant one (for convenience), as it is done within the MP3-code.



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Coarse to fine I

Given a piece of music we can take a recording, and this will be up to some frequency. The built-in microphone of a notebook or smart-phone will be able to record up to some frequency.
A better microphone (as I use it for demonstrations) allows show better and higher frequencies. I can record just a few seconds, or half an hour, I can try to record up to 20 kHz and with 8-bit quantization. According to WIKIPEDIA *many recording studios record in 24-bit/96 kHz (or higher) pulse-code modulation (PCM)* and use downsampling to 44100 samples per second for CD-production. But high precision devices can even record up to 192 kHz.
So practically speaking SOUND is something that can be measured (given suitable technology) up to any (reasonable) frequency.

Entities having a Spectrogram

Ignoring for a moment the difference between discrete and continuous (since we work at high resolution anyway, and in the limit the difference disappears, according to Riemann) we can thus certainly say: For any continuous and bounded function $h \in \boldsymbol{C}_b(\mathbb{R})$ we can produce a (bounded) spectrogram.

So how can we define the most general objects, which have a bounded spectrogram??

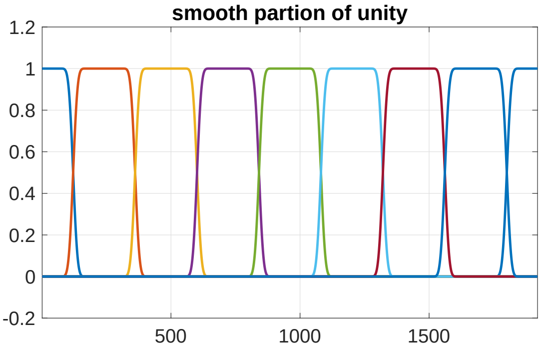
Of course we have to define Cauchy-sequences in $\boldsymbol{C}_b(\mathbb{R})$ (in the sense of corresponding spectrograms), and then an equivalence relation on them.

Analogy to Image Processing

Let us shortly mention the analogue to image processing, where we have seen in the last 2 decades how things develop.
Coming from analogue photography it was natural *to model images* as continuous, real-valued functions over a rectangular domain (say). With digital photography coming up it is more natural to think of images as pixel images, as they are downloadable from our cameras and mobile phones into MATLAB. But already technology requires to rescale to HDTV or your mobile phone, when it comes to the display of images, or transmission. Hence an image (in the idealistic sense) is somehow that **collection of all possible digital approximations** of the "true image". But isn't that another way to talk about

"Cauchy nets of digital images"?

Before doing so, we should (for a moment) fix our method of spectrograms, using smooth BUPUs such as:



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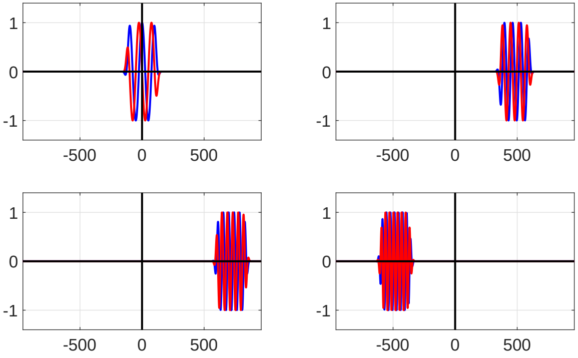

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Distributional Completion I

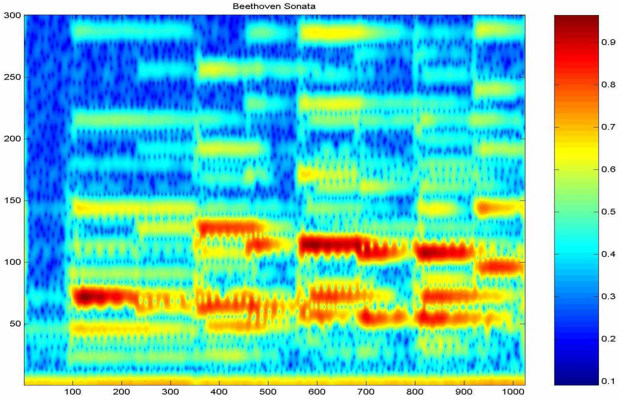
Starting from $\mathbf{C}_b(\mathbb{R})$ we can define (using Riemannian integrals) the spectrogram or STFT (we write $V_g(h)$ for the short-time Fourier transform of $h \in \mathbf{C}_b(\mathbb{R})$, with respect to the compactly supported (symmetric, and smooth) window g.
Recall that for a point $\lambda = (t, s) \in \mathbb{R} \times \widehat{\mathbb{R}}$ (phase space) we have

$$V_g(h)(\lambda) = \int_{-\infty}^{\infty} h(x)g(x-t) \cdot e^{-2\pi i \omega x} dx = \langle h, M_\omega T_t g \rangle.$$

The mapping $h \mapsto V_g(h)$ is a bounded mapping from $(\mathbf{C}_b(\mathbb{R}^d), \|\cdot\|_\infty)$ into $(\mathbf{C}_b(\mathbb{R}^{2d}), \|\cdot\|_\infty)$ (here $d = 1$), which is in fact injective.
Hence we can endow $\mathbf{C}$ with the norm $h \mapsto \|V_g(h)\|_\infty$, but unfortunately this space is not complete.

A Typical Musical STFT

A typical waterfall melody (Beethoven piano sonata) using the spectrogram: energy distribution in the TF = time-frequency plane:



continued

As we shall show there are different ways of completing. It is not enough to just take the completion with respect to the norm $h \mapsto \|V_g(h)\|_\infty$!
From an abstract point of view the w^*-completion is the correct one.
In fact, it raises the question (not relevant for this talk): What is the closure of $\boldsymbol{C}_b(\mathbb{R}^d)$ within $(\boldsymbol{S}_0'(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0'})$?
Similar: What is the closure of $\boldsymbol{C}_{ub}(\mathbb{R}^d)$ within $(\boldsymbol{S}_0'(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0'})$?
More: What is the closure of those functions from $L^\infty(\mathbb{R}^d)$ which vanish at infinity?

Equivalence Classes of mild Cauchy sequences I

Definition

A sequence $(h_n)_{n\geq 1}$ in $\mathbf{C}_b(\mathbb{R}^d)$ is called a **mild Cauchy sequence** if it is bounded in the STFT-sense, i.e.

$$\sup_k \|V_g(h_n)\|_\infty < \infty,$$

and moreover, for each $\lambda = (t, \omega)$ in $\mathbb{R}^d \times \widehat{\mathbb{R}}^d$ (phase space) one has a pointwise Cauchy-condition.

One can show that for any "mild Cauchy-sequence" (MCS) (h_n) the corresponding short-time Fourier transforms converge (pointwise) and define a bounded and continuous function.

Equivalence Classes of mild Cauchy sequences II

Of course, the same (bounded and continuous) function on $\mathbb{R}^d \times \widehat{\mathbb{R}}^d$ can be obtained in many different ways, and hence we have to introduce a concept of (mild) equivalence.
Either we define equivalence by saying that the (obviously existing) pointwise limits

$$H(\lambda) := \lim_{n\to\infty} V_g(h_n)(\lambda)$$

coincide for two different MCS, for any $\lambda \in \mathbb{R}^d \times \widehat{\mathbb{R}}^d$.
In this way we have a well-defined mapping from this new space of equivalence classes to the bounded and continuous functions, which can in fact be shown to be isometric (but certainly not surjective).

But how can one make these objects concrete?

First let me list a number of obstacles and problems, that one may have to overcome:

1. It is not convenient in practice to work with equivalence classes, can we find nice representatives?
2. What about the dependence on the given BUPU?
3. Can we, perhaps, also use Gaussian bump functions (as suggested by Gabor), and get the same collection?
4. Can we interpret these conditions as membership in some (infinite-dim.) Banach spaces?
5. What is the relationship to the classical theory of tempered distributions?

Why Functional Analysis and Function Spaces

In a first step let us - despite the concerns just mentioned - put the conditions in the context of Banach spaces.
We have a set of conditions (in principle a uniform boundedness of certain linear functionals), and hence it is plausible to find out, whether the collection of “signals” satisfying these conditions might be just a dual space.
This is in fact true, and give those abstract objects a concrete meaning as linear functionals, even endowed with a norm, a natural concept of w^* resp. distributional convergence and much more.

An Atomic Predual Space

Given the fact that we assume bounded output on what we can now call the "atoms" (TF-shifted version of our basic window g) we can easily derive that the problem can be linearized by considering the smallest Banach space containing all these atoms as a bounded subset, namely

$$AT(g) := \{f \mid f = \sum_{k=1}^{\infty} c_k M_{s_k} T_{x_i} g, \text{ with } (c_k) \in \ell^1\},$$

endowed with the natural inf-norm, turning $AT(g)$ into a Banach space.

Let us write $AF(g)$ for the "atomic functionals", i.e. the dual space of $AT(g)$.

Linearized Version

The key observation for us is now the following

Lemma

The mapping $(g, \sigma) \mapsto V_g(\sigma)$, *given by*

$$V_g(\sigma)(x, \omega) = \sigma(M_\omega T_x g), \quad x, \omega \in \mathbb{R}^d$$

defines a bilinear mapping: $AT(g) \times AF(g) \to \boldsymbol{C}_b(\mathbb{R}^{2d})$, *of course bounded in the usual sense.*

`BUT AGAIN we are faced with the problem that all the ingredients appear to depend on the choice of` g`!`

Fortunately this is not the case! Even if the verification of this claim requires some more work - not shown here - it is an important if one wants to rebuilt the theory from scratch.

Levels of Arguments

We have to argue at three different levels, when we want to provide good and well usable tools to the applied scientists.

- In order to be sure that we can understand all the concepts behind the scene we can use any available mathematical tool, including the most advanced (but well established) ones, or develop new arguments, whenever necessary.
- When it comes to the explanation of the proposed concepts and tools it should be done in a down to earth fashion, avoiding unnecessary difficulties (e.g. topological vector space, theory of nuclear Frechet spaces, etc.).
- Even if the engineers may not be able to follow the arguments it is important to have arguments that they do not get a (potentially flawed) toy-version of the actual theory, but a stream-lined and user-friendly tool that they can rely on.

Levels of Arguments II

The situation can be compared with situations in real life:

- We would like to have a safe car which is easy to use, without knowing how it works in detail.
- We want to be sure that the airplane is doing a good job in transporting us from A to B.
- In each case we may know the underlying principles in general, but we rely on the fact that certain agencies have checked that the technical object in use is satisfying all the requirements which are standard (state of the art).
- Along with the standards (and quality control) modern systems also provide consumer reports, user-feedback, and best practice reports. This is an area which is not even in discussion within the mathemtical sciences, except for algorithmic, i.e. in the context of numerical analysis.

More Examples from Daily Life

1. Mixer: should work in a reliable and efficient way, and even if people try to use it in a wrong way it should not do harm to the (naive) user!
2. A gas-oven should be convenient to use, but since it is potentially highly risky a lot of safety precautions have to be undertaken. Similar with camping cookers. The producer cannot rely on the technical knowledge of the user!
3. A cable car has to be safe, even under difficult, stormy and icy conditions. The user does not have to know anything to feel comfortable, but he should know that there are not just safe and non-safe cable cars (at least in Europe).
4. Even a simple city-bike has to satisfy certain standards, and people can use it after reading some **simple instructions**.
5. YOUR OWN EXAMPLE (and what does it tell you when you transfer it to the situation in mathematics!)

The Producers Perspective

In some sense we are producing mathematics still too much from a producers point of view. This means, that we rely on the usefulness of our results, we sharpen our tools and create a huge diversity of tools.
In fact, we have invented myriads of function spaces in order to study all possible operators, which may or may not be useful for applications. Sometimes a publication is motivated just by the question: Has anybody tried to use THIS operator on THAT function space, as obscure or perhaps mathematically difficult the question may be.
But exploring new paths to mountain peaks is not the same as developing a country and make it inhabitable!
So we have to bring in, at least as a *motivating aspect* the potential usefulness of a result, the easiness of use of a theorem, and not just its pure logical correctness.

A new approach to the SO-Banach-Gelfand Triple

With Mads S. Jakobsen we have a new approach to $(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0})$ starting from $(\boldsymbol{C}_0(\mathbb{R}^d), \|\cdot\|_\infty)$ and using BUPUs.

Assume that $\Psi = (\psi_k)_{k\in\mathbb{Z}^d}$ is such a BUPU on $\mathbb{R}^d$, with $\psi_k = T_{\alpha k}\psi_0$, for some smooth, compactly supported function. By decomposing a bounded and continuous function in $(\boldsymbol{C}_0(\mathbb{R}^d), \|\cdot\|_\infty)$ first on the time side, and then (after taking the Fourier Transform using the Riemann integral) on the Fourier transform side, we can define the predual in a *selective way*:

Definition

$$\boldsymbol{S}_0(\mathbb{R}^d) := \{f \in \boldsymbol{C}_0(\mathbb{R}^d) \mid \sum_n \sum_k \|\psi_n(\widehat{f\psi_k})\|_\infty < \infty\}$$

Many things to check (nothing is for free)

There are of course other alternatives, with different advantages or disadvantages. Let us mention a few of them here

1. The definition above does not require Lebesgue integration of the use of Plancherel's Theorem. On the other hand it is not obvious why the space defined in this way should be invariant under the Fourier transform;
2. Starting from $\left(\boldsymbol{L}^2(\mathbb{R}^d), \|\cdot\|_2\right)$ would make the definition more symmetric, but still the question is, whether (and why) changing the order of decomposition (time versus frequency side) does not matter.
3. In all the cases one has to check that different approaches define in the same space, with equivalence of norms.

BANACH GELFAND TRIPLES: a new category

Definition

A triple, consisting of a Banach space $\boldsymbol{B}$, which is dense in some Hilbert space $\mathcal{H}$, which in turn is contained in $\boldsymbol{B}'$ (hence w^*-dense there) is called a Banach Gelfand triple.

Definition

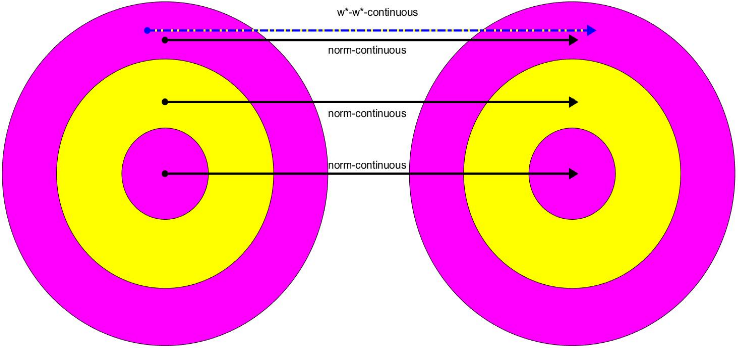
If $(\boldsymbol{B}_1, \mathcal{H}_1, \boldsymbol{B}_1')$ and $(\boldsymbol{B}_2, \mathcal{H}_2, \boldsymbol{B}_2')$ are Gelfand triples then a linear operator T is called a [unitary] Gelfand triple isomorphism if

1. A is an isomorphism between $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$.
2. A is a unitary isomorphism between $\mathcal{H}_1$ and $\mathcal{H}_2$.
3. A extends to a weak* isomorphism as well as a norm-to-norm continuous isomorphism between $\boldsymbol{B}_1'$ and $\boldsymbol{B}_2'$.

Banach Gelfand Triples

In principle every CONB (= *complete orthonormal basis*) $\Psi = (\psi_i)_{i \in I}$ for a given Hilbert space $\mathcal{H}$ can be used to establish such a unitary isomorphism, by choosing as $\boldsymbol{B}$ the space of elements within $\mathcal{H}$ which have an absolutely convergent expansion, i.e. satisfy $\sum_{i \in I} |\langle x, \psi_i \rangle| < \infty$.
For the case of the Fourier system as CONB for $\mathcal{H} = \boldsymbol{L}^2([0,1])$, i.e. the corresponding definition is already around since the times of N. Wiener: $\boldsymbol{A}(\mathbb{T})$, the space of absolutely continuous Fourier series. It is also not surprising in retrospect to see that the dual space $\boldsymbol{PM}(\mathbb{T}) = \boldsymbol{A}(\mathbb{T})'$ is space of *pseudo-measures*. One can extend the classical Fourier transform to this space, and in fact interpret this extended mapping, in conjunction with the classical Plancherel theorem as the first unitary Banach Gelfand triple isomorphism, between $(\boldsymbol{A}, \boldsymbol{L}^2, \boldsymbol{PM})(\mathbb{T})$ and $(\ell^1, \ell^2, \ell^\infty)(\mathbb{Z})$.

Banach-Gelfand-Tripel-Homomorphisms



Sufficient conditions for BGT-automorphisms

A couple of basic facts resulting direct from the definition, combined with the general fact that the space of w^*-continuous linear functionals on a dual space $\boldsymbol{B}'$ coincides naturally with the original Banach space, gives the following facts.
We apply them to the concrete Banach Gelfand Triple $(\boldsymbol{S}_0, \boldsymbol{L}^2, \boldsymbol{S}_0')$.

- For every BGTr-mapping there is an adjoint BGTr-mapping T^*, and $T^{**} = T$ for each of them;
- If a bounded linear mapping on $\left(\boldsymbol{L}^2(\mathbb{R}^d), \|\cdot\|_2\right)$ leaves $\boldsymbol{S}_0(\mathbb{R}^d)$ invariant and so does T^* (the Hilbert adjoint), then T (and also T^*) define BGTr-homomorphism.

Among others this principle can be used to show that the even *fractional Fourier transforms* define (unitary) BGTr-automorphisms on $(\boldsymbol{S}_0, \boldsymbol{L}^2, \boldsymbol{S}_0')(\mathbb{R}^d)$.

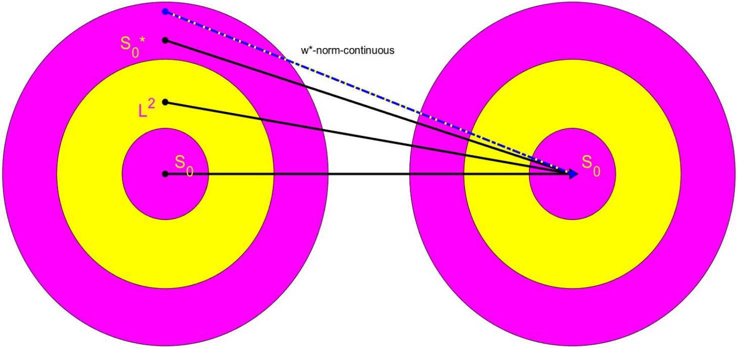
Some general scenarios arising in Gabor Analysis

There are situations, where one is forced to view even more general situations than BGTr-operators (BGOs), even if one is a priori interested only in the Hilbert space behaviour:
Here are some examples:

1. Let $g \in \boldsymbol{L}^2(\mathbb{R}^d)$ be given, and any lattice $\Lambda \lhd \mathbb{R}^d \times \widehat{\mathbb{R}}^d$. Then the coefficient operator
$$C : f \mapsto (V_g f(\lambda))_{\lambda \in \Lambda}$$
only maps $\boldsymbol{L}^2(\mathbb{R}^d)$ into $\mathrm{cosp}(\Lambda)$, but $\boldsymbol{S}_0(\mathbb{R}^d)$ into $\ell^2(\Lambda)$.
2. similar situation for the synthesis operator, which is its adjoint.
3. Finally the frame-operator is a priori an operator which maps $(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0})$ into $(\boldsymbol{S}_0'(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0'})$.

But this is good enough to show that it has a Janssen representation.

Regularizing Operators



Regularizers help to approximate distributions

An important family of regularizing operators are those bounded families of BGOs, where each of them maps $(\boldsymbol{S}_0'(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0'})$ into $(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0})$, and which form an approximating the identity. Abstractly speaking they form bounded nets of BGOs which converge strongly (e.g. in their action on $(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0})$) to the identity. Since we have

$$(\boldsymbol{S}_0' * \boldsymbol{S}_0) \cdot \boldsymbol{S}_0 \subset \boldsymbol{S}_0 \quad \text{and} \quad (\boldsymbol{S}_0' \cdot \boldsymbol{S}_0) * \boldsymbol{S}_0 \subset \boldsymbol{S}_0$$

this can be product-convolution (or convolution-product) operators.
A similar property is shared by Gabor multipliers with finitely many symbols (using lattice points in a bounded domain).

Justifying the Cauchy approach

With the help of regularizing operators, combined with the Banach-Steinhaus principle, it is not difficult to verify that there is actually a one-to-one correspondence (even in the sense of topological spaces) between the *equivalence classes of mild Cauchy-sequences* and the *dual space of* $\left(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0}\right)$.
First of all it is quite obvious that each mild Cauchy sequence defines (in its action on $\left(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0}\right)$) a ! bounded linear functional, i.e. some element $\sigma \in \boldsymbol{S}_0'$.
Conversely, one just has to apply any regularizing sequence (as described above) to $\sigma \in \boldsymbol{S}_0'$ in order to generate a mild Cauchy sequence. Of course the corresponding equivalence class allows to recover the original functional σ.

frametitle: bibliography I

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Some further books in the field are *in preparation*, e.g. on modulation spaces and pseudo-differential operators (Benyi/Okoudjou, Cordero/Rodino).

See also www.nuhag.eu/talks.