

## TEST I, Jan. 30th, 2008, 11:20 - 12:50: MATLAB, MCC3, Hans G. Feichtinger

Open a diary (and perhaps save it from time to time with the command "diary off; diary") preferably with your name, and don't forget to close it (and upload it) once you have finished your work. As far as I know new version overwrite the earlier ones, if they have the same name. The last one counts. 25 Pts. Max.

Please start with the standard test matrix  $T = \text{zeros}(:); T(:) = 1:9$ , and some random matrix  $A = 2 \cdot \text{rand}(5) - 1$  with random entries in the range of  $[-1, 1]$  for later use. Save the workspace at the end.

- 1: 4 Pts Generate a new matrix from  $A$  (call it  $B$ ) by replacing the third column of  $A$  by a linear combination of your choice of columns number 1 and 2. Determine the rank of the resulting matrix  $B$ , and describe the position of the pivot elements that such a matrix has! Find a basis or the row space of  $B$ , as rows of a new matrix  $BR$ . Check that these basis vectors are perpendicular to the null-space of  $T$ .
- 2: 3 Pts (a) Set up a consistent linear system by choosing as a right hand side for an equation of the form  $T * xt = bxt$ , by setting  $xt = [0, -2, 2]'$ . Calculate the right hand side  $bxt = T * xt$  and apply Gauss elimination to the expanded system  $[T, bxt]$  in order to verify that this system is of course consistent (has a solution). (b) Use PINV to determine a possible solution  $xx$  to the system. Check that this solution differs from  $xt$  only by an element of the null-space of  $T$ , as it should be.
- 3: 2 Pts Determine the coefficients of the polynomial  $p(x) = (1+x)^6$ , and remember, that the pointwise multiplication of polynomials can be realized using the CONV command to realize the coefficients of a product polynomial  $p(x) \cdot q(x)$ . (Maybe you recall PASCAL's triangle to check if your result is correct).
- 4: 3 Pts The matrix  $T$  has the remarkable (not very common) property that the row space and the column space are equal. Verify this claim by computing the projection matrix (of format  $3 \times 3$ ) onto both the column and the row space and verify that they are equal. [just a COMMENT: Such a property is true for any hermitian (resp. symmetric) matrix ( $T$  equals  $T'$ ), resp. normal matrices (by definition they a matrix is *normal* if one has  $T * T' == T' * T$ )]. Check that our matrix  $T$  is NOT normal!
- 5: 5 Pts Generate a matrix in the following form:  $D = \text{diag}([3, 1, 2])$ ,  $UU = \text{orth}(\text{rand}(3)); AA = UU * D * UU'$ ; and apply this matrix 12 times to a random vector of length one as a starting vector. After each application of the matrix  $AA$  normalize the result (i.e. make it a directional vector of length 1). Store the result and check the difference between those 12 vectors and their forerunners and display the size of those differences by a plot. Try to use a semi-logarithmic plot (semilogy).
- 6: 4 Pts Write a routine which creates a plot, with only one input parameter  $k$ , which plots the function  $x \mapsto x^k \cdot (1 + \cos(x)) \cdot \exp(x)$  in blue, on the interval  $[-\pi, \pi]$ . Find a good numerical approximation (3 digits) of its maximal value and the position where this maximal value is taken (the max. value for  $k = 2$  is about 18 and it is taken near the position 2.2). Mark this position in the plot with a red "\*". As time permits [2 extra-Pts] create from this a plot, showing the results for  $k = 2, 4, 6, 8$  in four windows within one figure.
- 7: 4 Pts In standard linear algebra courses we are told that a given set of vectors generating the vector space  $\mathbf{R}^n$  we can obtain a basis by starting from a set of vectors, and add linear independent vectors until we have a basis. At each step one has therefore to check whether the new vector is linear dependent from the previous ones. Do this numerically on a system of 7 column vectors in  $\mathbf{R}^5$ , which come in the form of a  $5 \times 7$  matrix  $H$ , with  $H = \text{rand}(5, 3); H3 = \text{rand}(5, 2); H2 = A * \text{rand}(3, 2); H = [H1, H2, H3]$ , . What do you observe (I do not need an M-file for this, just one concrete example, with a matrix  $BH$  as final result (test also that  $BH$  is indeed a basis for  $\mathbf{R}^5$ ).