

POLARISATION-ABERRATION RETRIEVAL FOR HIGH-NA SYSTEMS USING THE EXTENDED NIJBOER-ZERNIKE DIFFRACTION THEORY

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ABSTRACT

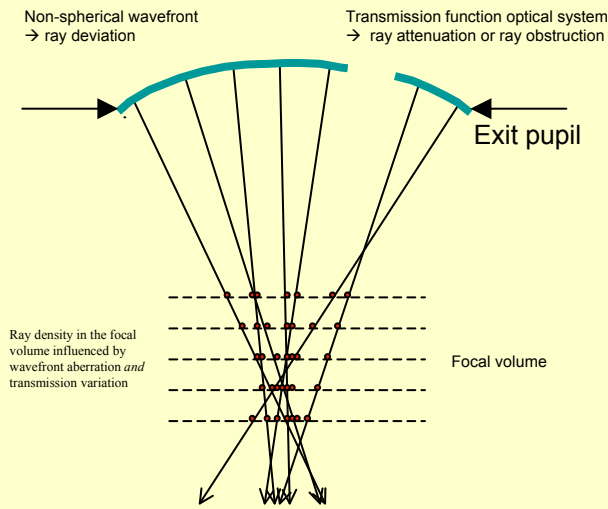
We have derived analytical expressions for the field components in the focal region of a high-numerical-aperture imaging system using the so-called Extended Nijboer-Zernike diffraction theory. It is shown that the transmission function, aberrations and polarisation properties of an imaging system with high numerical aperture can be derived from the through-focus intensity map via an inversion process based on this analysis.

Problem definition:

How to retrieve optical system properties (*amplitude, phase and polarisation* in the exit pupil) from *intensity* measurements through the *focal volume*?

1) Intuitive picture, based on ray optics:

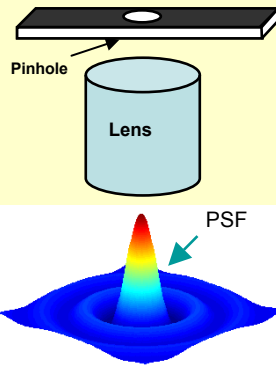
change in ray direction (wavefront aberration) and ray attenuation determine ray density (intensity) in focal volume !



2) More rigorous picture is based on scalar diffraction theory:

Huygens-Fresnel diffraction integral for light propagation from exit pupil → image plane.

In the presence of aberrations: theory of Nijboer-Zernike (1942)



Extension :
from source
to exit pupil
to focal volume :

Extended Nijboer-Zernike theory for through-focus point-spread function,

see: J. Opt. Soc. Am **A19**, 849-857 (2002)

Basic diffraction integral with defocus:
$$U(r, \varphi, f) = \frac{1}{\pi} \int_0^1 \rho \exp(i f \rho^2) \int_0^{2\pi} A(\rho, \vartheta) \exp\{i\Phi(\rho, \vartheta)\} \exp\{i 2\pi r \rho \cos(\vartheta - \varphi)\} d\vartheta d\rho$$

Introduction of Zernike polynomial expansion representing **amplitude and phase**:
$$A(\rho, \vartheta) \exp\{i\Phi(\rho, \vartheta)\} = \sum_{n,m} \beta_n^m R_n^{(m)} \exp(im\vartheta)$$

Bessel series solution:
$$U(r, \varphi, f) = 2 \sum_{n,m} i^m \beta_n^m V_n^{(m)}(r, f) \exp(im\varphi) \quad \text{with:} \quad V_n^{(m)}(r, f) = \exp(i f) \sum_{l=0}^{\infty} \left(\frac{-i f}{\pi r}\right)^l \sum_{j=0}^l u_{lj} \frac{J_{|m|+2j+1}(2\pi r)}{2\pi r}$$

3) Vector diffraction theory for high-NA focused beams (forward):

ENZ-theory for complex exit pupil function with due account of

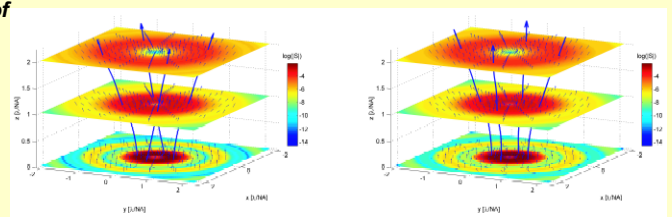
a) radiometric effect b) high-NA defocusing factor

c) polarisation state

Electric field density is:
$$w_E = \frac{\epsilon_0}{4} n_r^2 |E|^2 = \frac{\epsilon_0}{4} n_r^2 \{E_1 E_1^* + E_2 E_2^* + E_3 E_3^*\}$$

with explicit analytic expressions available for electric field vector components in the focal volume.

See: J. Opt. Soc. Am. **A20**, 2281-2292 (2003)



Vectorial **forward** calculation using the Extended Nijboer-Zernike theory. Energy flow (arrows) and intensity distribution (colour-coded) in the focal region.

Left-hand figure: circularly polarised; Right-hand figure: circularly polarised + orbital angular momentum.

The lateral coordinates are in units λ/NA . The light propagates in the vertical direction; the NA of the focused beam is 0.85

4) Polarisation-aberration retrieval using ENZ-theory at high NA:

'backward' calculation from energy density in focal volume leads to complex lens function + polarisation effects (birefringence)

State of polarisation in the exit pupil depends on :

- a) geometrical lens properties (NA, transmission, aberration),
- b) birefringence ('scrambling' of polarisation state)

Description of state of polarisation via Jones matrix :

$$\begin{pmatrix} E_{xj} \\ E_{yj} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} a_j \\ b_j \end{pmatrix} \text{ the incident polarisation.}$$

For a pure phase birefringence, the Jones matrix reduces to :

$$\begin{pmatrix} E_{xj} \\ E_{yj} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ -m_{12}^* & m_{11}^* \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} \quad \text{with} \quad |m_{11}|^2 + |m_{12}|^2 = 1$$

Conclusion:
four 'inverse' operations are sufficient for retrieval of the 'polarisation-aberrations' of an optical imaging system

Detailed information about the ENZ-theory and its applications in optical aberration theory, lithography and lens metrology can be found at the website:

<http://www.nijboerzernike.nl>