Extended Nijboer-Zernike (ENZ) based evaluation of amplitude and phase aberrations on scaled and annular pupils

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Summary

Based on a recently developed formalism to describe pupil scaling in terms of Zernike coefficients, we present an ENZ-retrieval method for aberrated pupil functions on an annular set from through-focus intensities.

Introduction and results

The operation of pupil scaling occurs in several fields in optics, such as advanced lithography and ophthalmology, [1]-[2]. Recently [3], the operation of pupil scaling has been concisely formulated in terms of linear transformations of the Zernike coefficients of the pupil function at full scale. This gives rise to an ENZ-type computation scheme for the through-focus, complex point-spread function corresponding to the scaled pupil function in terms of the complex Zernike coefficients of the pupil function at full scale. The basic functions of the full scale ENZ-method, which are the complex point-spread functions arising from the Zernike terms $R_n^m(\rho)\cos m\theta$, only need to be adjusted in a simple and explicit manner. In a similar manner, the ENZ-method can be adapted so as to produce the through-focus complex point-spread function of an aberrated pupil function vanishing outside an annular set.

A salient feature of ENZ-theory is its ability to retrieve pupil functions from the through-focus intensity point-spread function, see, for instance [4]. Under a "small-aberrations" assumption, this is based on linearizing the intensity point-spread function of the ENZ-theory, and estimating the Zernike coefficients of the pupil by optimizing the match between the measured intensity data and the linearized ENZ-intensity. In this presentation, we show how the ENZ-retrieval formalism must be adjusted so as to achieve pupil reconstruction from intensities generated by aberrated pupils on an annular set. We show retrieval results for a Gaussian, comatic pupil function on an annulus $\frac{1}{2} \le \rho \le 1$ with ρ the normalized radial variable.

Scaling operation in terms of Zernike coefficients

We assume that the pupil function $P(\rho,\theta)$ is symmetric in θ so that it allows a Zernike expansion

$$P(\rho,\theta) = \sum\nolimits_{n,m} \beta_n^m R_n^m(\rho)\cos(m\theta) \tag{1}$$

with $R_n^m(\rho)$ the (radial part of the) Zernike polynomials. According to [3], the scaled pupil function $P(\varepsilon\rho,\theta)$, with ε the scaling parameter, has the Zernike expansion

$$P(\varepsilon\rho,\theta) = \sum\nolimits_{n,m} \beta_n^m(\varepsilon) R_n^m(\rho)\cos(m\theta) \tag{2}$$

in which

$$\beta_n^m(\varepsilon) = \sum_{n'} \left(R_{n'}^{n}(\varepsilon) - R_{n'}^{n+2}(\varepsilon) \right) \beta_{n'}^m , \tag{3}$$

where the summation is over *n'=n,n+2,…* .

Scaling operations and point-spread functions

According to the ENZ-theory, the point-spread function $U(r,\varphi,f)$ corresponding to $P(\rho,\theta)$, with r,φ the spatial variables in polar coordinates in the image plane with defocus parameter f, is given by

$$\begin{aligned} U(r,\varphi,f) &= \tfrac{1}{\pi}\int_0^1\int_0^{2\pi} P(\rho,\theta) e^{if\rho^2} e^{2\pi i\rho r\cos(\theta-\varphi)} \rho d\rho d\theta \\ &= 2\sum_{n,m} i^m \beta_n^m V_n^m(r,f)\cos(m\varphi). \end{aligned} \tag{4}$$

Here the terms

$$V_n^m(r,f) = \int_0^1 e^{if\rho^2} R_n^m(\rho) J_m(2\pi\rho r)\rho d\rho \tag{5}$$

can be evaluated according to the semi-analytic expressions developed in [5,6].

The point-spread function $U_\varepsilon(r,\varphi,f)$ of the pupil function with NA reduced to a fraction ε of the pupil size is given by

$$\begin{aligned} U_\varepsilon(r,\varphi,f) &= \tfrac{1}{\pi}\int_0^\varepsilon\int_0^{2\pi} P(\rho,\theta) e^{if\rho^2} e^{2\pi i\rho r\cos(\theta-\varphi)} \rho d\rho d\theta \\ &= 2\sum_{n,m} i^m \beta_n^m V_n^m(r,f;\varepsilon)\cos(m\varphi) \end{aligned} \tag{6}$$

in which

$$V_n^m(r,f;\varepsilon) = \varepsilon^2 \sum_{n'} \left(R_n^{n'}(\varepsilon) - R_n^{n'+2}(\varepsilon) \right) V_{n'}^m(r\varepsilon, f\varepsilon^2), \tag{7}$$

where the summation is over *n'=m,m+2, … , n*.
For details, see [7].

Through focus point-spread function of a pupil on an annular set

Assume we have a pupil function $P(\rho,\theta)$ that vanishes outside the set $\varepsilon \le \rho \le 1$. On this set $\varepsilon \le \rho \le 1$, there is a Zernike representation (possibly non-unique, but well converging)

$$P(\rho,\theta) = \sum_{n,m} \gamma_n^m R_n^m(\rho)\cos(m\theta). \tag{8}$$

The through-focus point-spread function of this P is given as

$$U(r,\varphi,f) = 2\sum_{n,m} i^m \gamma_n^m W_n^m(r,f;\varepsilon)\cos(m\varphi), \tag{9}$$

where

$$W_n^m(r,f;\varepsilon) = V_n^m(r,f) - V_n^m(r,f;\varepsilon) \tag{10}$$

with V_n^m functions given in (5) and (7). See [7] for details.

ENZ-retrieval for pupils on an annular set

Assume we have a measured through-focus intensity I_{meas} corresponding to an aberrated pupil function P on an annular set $\varepsilon \le \rho \le 1$. Proposing $P(\rho,\theta)$ of the form (8), with γ_n^m to be estimated, we match I_{meas} with the intensity

$$I(r,\varphi,f) = \left|U(r,\varphi,f)\right|^2 = \left|2\sum_{n,m} i^m \gamma_n^m W_n^m(r,f;\varepsilon)\cos(m\varphi)\right|^2 \tag{11}$$

from the ENZ-theory by optimizing the γ's.

Example

Consider the case that

$$P(\rho,\theta)=e^{-\gamma\rho^2+i\alpha R_3^1(\rho)\cos\theta} \quad , \quad \tfrac{1}{2}\le\rho\le 1\ , \quad 0\le\theta\le 2\pi, \tag{12}$$

with $\gamma=\alpha=0.1$. We compute the "measured" through-focus intensity I_{meas} in simulation, and we apply the above given retrieval procedure. The totality of all γ_n^m in (11) is small compared to $\frac{3}{4}\approx\gamma_0^0$ (the coefficients of the aberration-free term W_0^0 in Eq.(11)). The matching procedure is iterative in which the $|\ \ |^2$ in (11) is expanded and the two γ's pertaining to small cross-terms with $(m_1,n_1)\ne(0,0)\ne(m_2,n_2)$ are replaced by their values in iteration $k-1$. Then the $\gamma_n^m(k)$ are found by minimizing the mean square difference of the measured intensity I_{meas} and the approximated right-hand side of (11) that involves γ_n^m in a linear way. For more details, see [7]. We have worked this out for the even more complicated case that the system has a high numerical aperture (NA=0.95). In Fig. 1 we present the results of this retrieval operation at high numerical aperture for an annular aperture with *ε=0.50*. The upper left graph shows the modulus of the difference between the analytic pupil function $P_{analytic}$ and the pupil function P_{beta} following from our Zernike expansion up to a certain chosen degree.

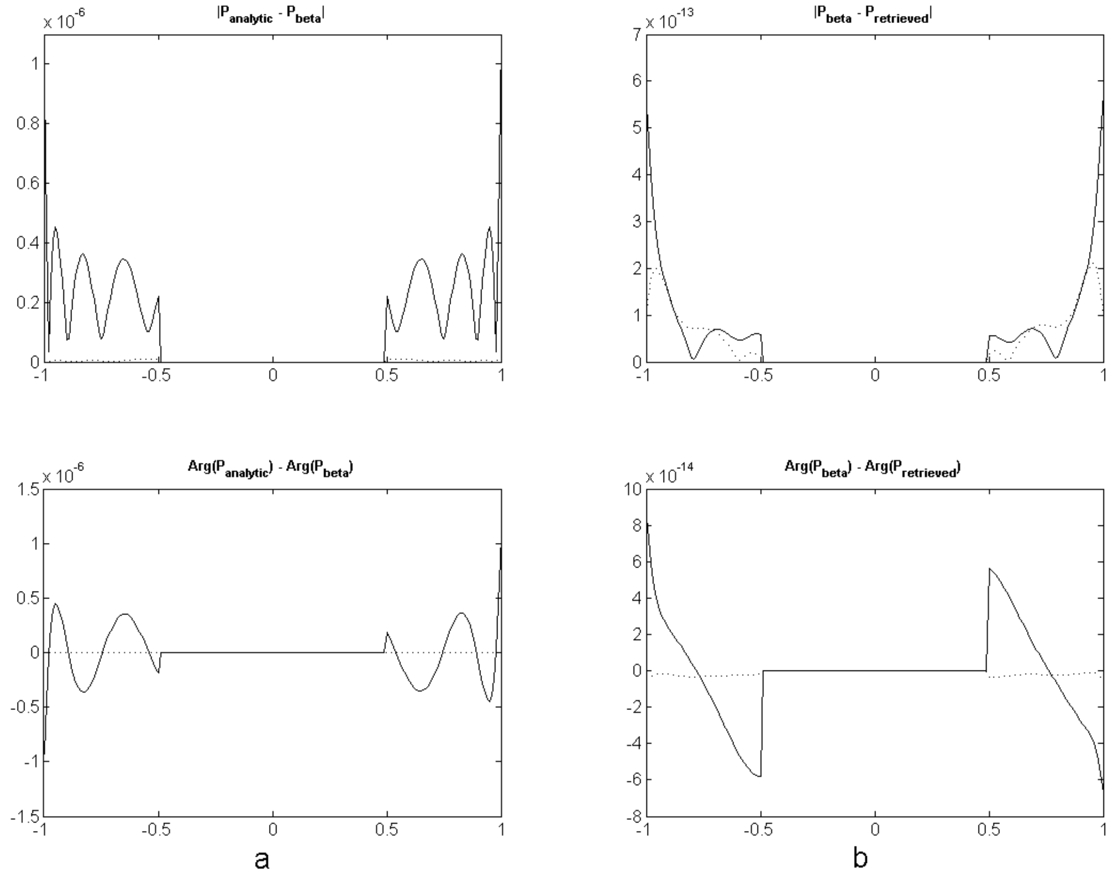

Fig 1. Pupil function reconstruction using the ENZ-retrieval method for a Gaussian, comatic pupil, see (12), with *γ=0.1* and α_3^1*=0.1* on an annular set $\varepsilon=\frac{1}{2}\le\rho\le 1$.

The lower left graph presents the difference in argument between the analytic and Zernike-fitted function. The maximum degree of the Zernike expansion has been chosen so that the maximum differences are of the order of 10^{-6}. The two right-hand graphs (Fig. 1b) present the differences in modulus and argument of the Zernike-fitted pupil function P_{beta} and the retrieved pupil function $P_{retrieved}$ according to our linearized inversion scheme with successive iterations, see also [8] for the high-numerical-aperture case.

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General information on the Extended Nijboer-Zernike diffraction theory and pupil retrieval method can be found on the website: http://www.nijboerzernike.nl.