Strobl07

Trends in Harmonic Analysis
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Talks/Posters :: Abstracts

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1 Invited Talks

1.1 Pete Casazza

Department of Mathematics, University of Missouri, UNITED STATES

The Feichtinger Conjecture in Mathematics and Engineering

We will see that the Feichtinger Conjecture in frame theory is equivalent to fundamental unsolved problems in a dozen areas of research in pure mathematics, applied mathematics and engineering including the Kadison-Singer Problem, the Bourgain-Tzafriri Conjecture, the Paving Conjecture etc. This gives all these research areas common ground on which to interact as well as explaining why each of them has volumes of literature on their respective problems without a satisfactory resolution. We will look at some of the equivalences of the Feichtinger Conjecture in operator theory, Banach space theory, harmonic analysis, and applied math/engineering.

1.2 Albert Cohen

Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie, FRANCE

Near optimal recovery of arbitrary signals from uncomplete measurements

Compressed sensing is a recent concept in signal and image processing where one seeks to minimize the number of measurements to be taken from signals or images while still retaining the information necessary to approximate them well. The ideas have their origins in certain abstract results from functional analysis and approximation theory but were recently brought into the forefront by the work of Candes-Romberg-Tao, and Donoho who constructed concrete algorithms and showed their promise in application. There remain several fundamental questions on both the theoretical and practical side of compressed sensing. This talk is primarily concerned about one of these issues revolving around just how well compressed sensing can approximate a given signal from a given budget of fixed linear measurements, as compared to adaptive linear measurements. More precisely, we consider discrete N-dimensional signals x with $N >> 1$, allocate $n << N$ linear measurements of x, and we describe the range of k for which these measurements encode enough information to recover x to the accuracy of best k-term approximation. We also consider the problem of having such accuracy only with high probability.

1.3 Vladimir Temlyakov

Department of Mathematics, University of South Carolina, UNITED STATES

Convergence of Greedy Approximation for the Trigonometric System

We study the following nonlinear method of approximation by trigonometric polynomials. For a periodic function f we take as an approximant a trigonometric polynomial of the form $G_m(f) := \sum_{k\in\Lambda} \hat{f}(k)e^{i(k,x)}$, where $\Lambda \subset Z^d$ is a set of cardinality m containing the indices of the m biggest (in absolute value) Fourier coefficients $\hat{f}(k)$ of function f. Note that $G_m(f)$ gives the best m-term approximant in the L_2-norm and, therefore, for each $f \in L_2$, $\|f - G_m(f)\|_2 \to 0$ as $m \to \infty$. It is known from previous results that in the case of $p \neq 2$

the condition $f \in L_p$ does not guarantee the convergence $\|f - G_m(f)\|_p \to 0$ as $m \to \infty$. We study the following question. What conditions (in addition to $f \in L_p$) provide the convergence $\|f - G_m(f)\|_p \to 0$ as $m \to \infty$?

see also: http://univie.ac.at/nuhag-php/dateien/talks/671_Wconf.pdf

1.4 Roman Vershynin

Department of Mathematics, University of California, Davis, UNITED STATES

Uncertainty principles, frames and vector quantization

The uncertainty principle in harmonic analysis has been interpreted and enriched in a way to yield algorithms for signal reconstruction (sparse recovery). We will see a new application of the uncertainty principles – robust vector quantization. For frames in N dimensions that satisfy the Uncertainty Principle, one can quickly convert every frame representation into a more regular Kashin's representation, whose coefficients all have the same magnitude. Information tends to spread evenly among these coefficients. These representations have great error reduction power. In particular, scalar quantization of Kashin's representations yields robust vector quantizers. Joint work with Yuri Lyubarskii.

2 Talks

2.1 Anvarjon Ahmedov

Faculty of computer technology and community, Kyrgyz-Uzbek University, KYRGYZSTAN

Spectral expansions of pseudodifferential operators

2.2 Jean-Pierre Antoine

Institut de Physique Théorique, Université catholique de Louvain, BELGIUM

Locally supported orthogonal wavelet bases on the sphere via stereographic projection

The stereographic projection determines a bijection between the two-sphere, minus the North Pole, and the tangent plane at the South Pole. This correspondence induces a unitary map between the corresponding L^2 spaces. As it is known, this map in turn leads to an equivalence between the continuous wavelet transform formalisms on the plane and on the sphere. More precisely, any plane wavelet may be lifted, by inverse stereographic projection, to a wavelet on the sphere. In this work, we apply this procedure to orthogonal bases of locally supported wavelets in the plane and get similar bases on the sphere. Numerical examples are given and compared with the results obtained by other types of spherical wavelet frames.

2.3 Ataollah Askari Hemmat

Department of Mathematics, Vali Asr University, Rafsanjan, IRAN

Fractal sets and their relation with wavelet sets

Sets with non-integeral Hausdorff dimension are called fractals by Mandelbrot, when they have the additional property of being in some sense either strictly or statistically self similar, have been used to model various physical phenomena. In this article we will review the theory of wavelet sets and fractls. In dimension 2 we will prove that under certain conditions the product of two sets is a wavelet set. This is completely new and we will present few new examples. We will explain fractal sets and show how some fractal sets can be obtained from wavelet sets.

Keywords: Fractals, Multiresolution Analysis, Wavelets, Wavelet Sets, Digits.

2.4 Agissilaos Athanassoulis

Department of Naval Architecture and Marine Engineering, National Technical University of Athens, GREECE

Generalization of the Weyl-Moyal calculus and computational homogenization of wave propagation

In the study of wave propagation over distances much longer than the typical wavelength phase-space methods are often used, i.e. spectral densities are used as a homogenized representation of the wavefield, and kinetic equations are constructed for their evolution.

The Wigner Transform (WT) is a nonlinear, nonparametric spectral density, first introduced by E. Wigner in the context of quantum mechanics. Recently it has been extensively used in the formulation of phase-space models for a variety of problems, such as geometrical optics limits, periodic problems, nonlinear and/or random waves. Physical areas of application include semiconductors, linear and nonlinear optics, water waves and more.

However, the WT features counterintuitive interference components, which make computation and interpretation problematic. To face this, variants such as Wigner measures are typically used in practice.

We use smoothed Wigner Transforms (SWT) to study wave propagation. We present the derivation of new, to the best of our knowledge, exact equations of motion for the SWT covering a broad class of wave propagation problems. As a special case we get exact equations of motion for spectrograms of wave fields. These equations are typically pseudodifferential.

The new equations are used for the construction of a reliable, efficient, homogenized (slow-scale) numerical solver for the Schrodinger equation. The new solver successfully captures the spatial structure of caustics, while clearly outperforming conventional, full (i.e. not homogenized) solvers. A subtle issue which we discuss is the construction of an appropriate 'slow-scale error', i.e. a quantitative measure of how good an approximation our SWT solution offers; exact solutions are used to test our 'slow-scale error', with good results.

2.5 Damir Bakić

Department of Mathematics, University of Zagreb, CROATIA

Orthonormal wavelets with general integer dilations

In this talk we shall discuss a method for constructing orthonormal wavelets in $L^2(R^n)$ with dilations induced by expanding integer matrices of arbitrary determinant. In particular, we are interested in scaling sets and, more generally, in orthonormal wavelets that arise from generalized multiresolution analyses with singly generated core spaces. Several examples will be included. The talk is based on the results of a joint research with Edward N. Wilson.

2.6 Peter Balazs

Acoustics Research Institute / NuHAG, AUSTRIA

Weighted and controlled frames

Weighted and controlled frames have been introduced recently to improve the numerical efficiency of iterative algorithms for inverting the frame operator. In this talk we develop systematically these notions including their mutual relationship. We will show that controlled frames are equivalent to standard frames and so this concept gives a generalized way to check the frame condition. We investigate the numerical advantages of introducing these concepts in the sense of preconditioning. We consider the special case of semi-normalized weights, where the concepts of weighted frames and standard frames are interchangeable. We also make the connection with frame multipliers.

2.7 Pawel Bechler

Institute of Applied Mathematics and Mechanics, Faculty of Mathematics, Informatics and Mechanics, Warsaw University, POLAND

Frames and n-term approximation

In this presentation the properties of n-term frame approximation will be considered in a general setting. Specifically, I would like to consider the question how basis-specific notions like greedy/ quasi greedy / almost-greedy (basis), which describe the behavior of bases with respect to n-term approximation, can be transfered to the world of frames.

2.8 Jeffrey Blanchard

Washington University in St. Louis, UNITED STATES

Minimally supported frequency composite dilation wavelets

The system $\Psi = (\psi^1, \psi^2, \ldots, \psi^L)^T \subset L^2(\mathbb{R}^n)$ is an (AB, Γ)-*Composite Dilation Wavelet* if $\{D_a^j D_b T_k \psi^i : j \in \mathbf{Z}, b \in B, k \in \Gamma, i = 1, \ldots, L\}$ is an orthonormal basis for $L^2(\mathbb{R}^n)$, where $A = \{a^j : j \in \mathbb{Z}\}$ is a group generated by an expanding matrix, a, B is a subgroup of $GL_n(\mathbb{R})$, and Γ is a full rank lattice. Given a finite group B, we present admissibility conditions for arbitrary lattices and then for arbitrary expanding matrices. We show that these admissibility conditions are sufficient to generate minimally supported frequency, (AB, Γ)-composite dilation wavelets for $L^2(\mathbb{R}^n)$. We then show that for any finite group B whose fundamental region is bounded by hyperplanes through the origin, such as Coxeter groups or rotation groups, we can always find admissible lattices and expanding matrices. Given the existence of MSF, composite dilation wavelets for $L^2(\mathbb{R}^n)$, we explore ideas to minimize the number of wavelet generators, L, for the system Ψ. We will present examples of singly generated composite dilation wavelets for $L^2(\mathbb{R}^n)$ and examine how reducing the generators limits our freedom.

2.9 Ole Christensen

Department of Mathematics, University of Denmark, DENMARK

Pairs of explicitly given Gabor frames

We provide a construction of explicitly given pairs of Gabor frames. The window function and the dual window have compact support and can be chosen with polynomial decay of any desired order in the Fourier domain. In particular, the window function can be chosen as a B-spline, and the dual window as a finite linear combination of translates hereof.

2.10 Elena Cordero

Department of Mathematics, University of Torino, ITALY

Strichartz Estimates in Wiener Amalgam Spaces for the Schrödinger Equation

We study the dispersive properties of the Schrödinger equation. Precisely, we look for estimates which give a control of the local regularity and decay at infinity separately. The Banach spaces that allow such a treatment are the Wiener amalgam spaces, and Strichartz-type estimates are proved in this framework. These estimates improve some of the classical ones in the case of large time.

2.11 Stephan Dahlke

University of Marburg, GERMANY

Shearlet Coorbit Spaces and Related Banach Frames

In recent years, a lot of attempts have been made to extract directional information from images such as curvelets, ridgelets, contourlets and shearlets. Among these, the shearlet transform stands out since it is related with group theory, i.e., it stems from a square-integrable representation of a locally compact group, the shearlet group. This specific feature makes it possible to combine the shearlet approach with the coorbit space theory developed by Feichtinger and Grchenig. In this talk, we show that indeed all the assumptions needed to apply the coorbit theory can be satisfied. We establish the corresponding smoothness spaces, the shearlet coorbit spaces, and we explain how the representation can be discretized in order to obtain atomic decompositions and Banach frames for these new smoothness spaces.

(joint work with G. Kutyniok, G. Steidl and G. Teschke)

2.12 Maurice De Gosson

NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

The Weyl Representation of Metaplectic Operators and the Conley-Zehnder Index

The metalectic group plays an important role in time-frequency analysis, in quantum mechanics, and in representation theory. We identify the Weyl symbol of metaplectic operatorsusing the notion of symplectic Cayley transform, and show that the correct phase is obtained by using the Conley-Zehnder index familiar from the theory of periodic Hamiltonian orbits.

2.13 Maarten V. De Hoop

Purdue University, UNITED STATES

Dyadic parabolic decomposition, approximation of functions by wave packets, and evolution equations

We discuss transforms in the family of curvelets and their discretization, numerically honoring the necessary decay estimates in phase space, in the context of constructing solutions to evolution equations of limited smoothness. We present the concentration of curvelets by the associated solution operator. We then discuss an approximation of functions following the dyadic parabolic decomposition that leads to sparse representations of initial data and solutions irrespective of scale.

The is joint research with F. Andersson, M. Carlsson, H. Smith and G. Uhlmann.

2.14 Mohammad Ali Dehghan

Department of mathematics, Vali-Asr University, Rafsanjan, IRAN

Biframe Duals and Reflexivity

Biframe Duals and Reflexivity

M.A.Dehghan and S.Eismaiilpour

The Ron-Shen duality priciple , Wexler-Raz biorthogonality relations and other duality priciples play a fundamental role for analyzing dual Gabor frames.The question arises wether which of priciples can be generalized to abstract theory. We are going to show that for each sequence in a separable Hilbert space we can construct a corresponding sequence with a kind of duality relation between them.One type of this construction has done by P.G.Casassa, G.Kutyniok and M.C.Lammers by use two orthonormal bases. We extend this construction by use of two frames instead of orthonormal bases. These duals depend on the situation of two frames. In fact the independence of the situation of frames leads to reflexive sequence that has studied in the last of article.

2.15 Victor Didenko

Department of Mathematics, Universiti Brunei Darussalam, BRUNEI DARUSSALAM

Estimates of the spectral radius for refinement and subdivision operators

The talk presents lower bounds for the spectral radii of refinement and subdivision operators with continuous matrix symbols and with dilations from a class of isotropic matrices. This class contains main dilation matrices used in wavelet analysis. After obtaining general formulas, two kinds of estimate for the spectral radii are established: viz. – estimates using point values of the symbols and others involving integrals of different dimensions over designated subsets of tori. For some symbol classes the exact value of the spectral radius of the refinement operator is found.

2.16 Mihaela Dobrescu

Christopher Newport University, UNITED STATES

Circular and Hyperbolic Rotations and MRA Wavelets

In this short presentation, we will discuss the interplay between finite Coxeter groups and the construction of wavelet sets, Multireslolution Analysis and sampling.

2.17 Martin Ehler

Philipps-University Marburg, GERMANY

N-term approximation by nonseparable wavelet bi-frames

First, we derive equivalences between the Besov norm and the sequence norm of wavelet bi-frame coefficients. Then, we determine the best n-term approximation spaces of multivariate wavelet bi-frame dictionaries. This talk generalizes dyadic results to isotropic dilation matrices.

2.18 Miroslav Englis

Mathematics Institute Zitna, Prague, CZECH REPUBLIC

Toeplitz operators and Segal-Bargmann analysis

2.19 Brendan Farrell

Math Department, University of California, Davis, UNITED STATES

Inverse-Closedness of a Banach Algebra of Integral Operators on the Heisenberg Group

We show that a class of integral operators on the Heisenberg group, given by the off-diagonal decay of the kernel, is inverse-closed. Using this result, we obtain a version of Wiener's lemma for convolution on the Heisenberg group. The same approach yields a similar result for a class of pseudodifferential operators.

2.20 Milton Ferreira

Universidade de Aveiro, PORTUGAL

Spherical Continuous Wavelet Transforms arising from sections of the Lorentz group

We consider the conformal group of the unit sphere S^{n-1}, the Lorentz group $Spin^+(1,n)$ (double covering group of the proper Lorentz group $SO_0(1,n)$) for the study of spherical continuous wavelet transforms (SCWT). The parameter space is determined by a factorization of the gyrogroup of the unit ball by an appropriate gyro-subgroup. We define admissible local and global sections and we study some families of sections that give rise to SCWT. These SCWT extend the SCWT of J.P. Antoine and P. Vandergheynst , which itself arises from the fundamental section of our homogeneous space. The advantage of these new SCWT is that they incorporate relativistic movements on the unit sphere which are suitable for signal processing on the unit sphere.

2.21 Massimo Fornasier

PACM-Princeton, UNITED STATES

The theory of iterative thresholding algorithms and their acceleration methods

Since the work of Donoho and Johnstone, soft and hard thresholding operators have been extensively studied for denoising of digital signals, mainly in a statistical framework. Usually associated to wavelet or curvelet expansions, thresholding allows to eliminate those coefficients which encode noise components. The assumption is that signals are sparse in origin with respect to such expansions and that the effect of noise is essentially to perturb the sparsity structure by introducing non zero coefficients with relatively small magnitude. While a simple and direct thresholding is used for statistical estimation of the relevant components of an explicitly given signal, and to discard those considered disturbance, the computation of the sparse representation of a signal implicitly given as the solution of an operator equation or of an inverse problem requires more sophistication. We refer, for instance, to deconvolution and superresolution problems, image recovery and enhancing, and problems arising in geophysics and brain imaging. In these cases, thresholding has been combined with classical Landweber iterations to compute the solutions. In this talk we present a general theory of iterative thresholding algorithms which includes soft, hard, and the so-called firm thresholding operators. In particular, we develop a unified variational approach of such algorithms which allows for a complete characterization of their convergence properties. As a matter of fact, despite their simplicity which makes them very appealing to users and their enormous impact for applications, iterative thresholding algorithms converges very slowly and might be impracticable in certain situations. By analyzing their typical convergence dynamics we propose acceleration methods based 1. on projected gradient iterations, 2. on alternating subspace corrections (domain decompositions.) Also for both these latter families of algorithms, a variational approach is fundamental in order to correctly analyse the convergence. The talk partially summarizes recent joint results with Ingrid Daubechies, Ron DeVore, Sinan Gunturk, and Holger Rauhut.

2.22 Hartmut Führ

Lehrstuhl A für Mathematik, RWTH Aachen, GERMANY

Painless Gabor expansions on homogeneous manifolds

We present a novel approach to the definition of Gabor systems on homogeneous manifolds with compact stabilizers. The associated Gabor transform has several attractive properties: It maps L^2-functions to continuous functions on the cotangent bundle, and preserves L^2-norms. Moreover, it is covariant with respect to the natural group action on the cotangent bundle. As an example, we present the case of the sphere in more detail.

2.23 Sadek Gala

University of Mostaganem, ALGERIA

Decomposition of distribution in BMO spaces

In this paper, we characterize f such that if the inequality

$$|\int\limits_{R^d} f.(u\nabla v - v\nabla u)dx|C\|u\|_H\|v\|_H$$

holds for all $u, v \in D(R^d)$, then f can be represented in the form

$$f = \nabla g + DivH$$

where $g \in BMO(R^d)$, H is a skew-symmetric matrix field such that $H \in BMO(R^d)^d$.

2.24 Daryl Geller

Department of Mathematics, Stony Brook University, NY, UNITED STATES

Nearly Tight Frames and Space-Frequency Analysis on Manifolds

We construct wavelet frames for smooth compact oriented Riemannian manifolds, which are nearly tight (that is, the ratio of the frame bounds can be made as close to 1 as one likes by adjusting the parameters), and for which one has a space-frequency analysis which is analogous to the usual time-frequency analysis on the real line. We also discuss characterizations of function spaces, through knowledge of only the size of frame coefficients. This is joint work with Azita Mayeli.

2.25 Smbat Gogyan

Institute of Mathematics, Polish Academy of Sciences, POLAND

Renormalized Haar system as quasi-greedy basis in $L^1(0,1)$

Let $\{h_n\}$ be the Haar system (normalized in $L^1(0,1)$). It is known that the Haar system is not quasi-greedy basis in $L^1(0,1)$, i. e. there exists $x = \sum_{n=1}^{\infty} c_n h_n \in L^1(0,1)$ such that

$$\sum_{n=1}^{\infty} c_{\varrho(n)} h_{\varrho(n)} \neq x,$$

where $\{c_{\varrho(n)}\}$ is any nonincreasing (by absolute value) rearrangement of $\{c_n\}$.

We prove that for any $0 < t < 1$ and $x = \sum_{n=1}^{\infty} c_n h_n \in L^1(0,1)$ there exists rearrangement of $\{c_n\}$ such that

$$\min_{1\le k\le n} \{| c_{\sigma(k)} |\} \ge t \max_{k>n} \{| c_{\sigma(k)} |\} \tag{1}$$

and

$$x = \sum_{n=1}^{\infty} c_{\sigma(n)} h_{\sigma(n)}.$$

On the other hand S. Konyagin and V. Temlyakov proved the following: if $\Psi = \{\psi_n\}$ is a quasi-greedy basis in a Banach space X, then for arbitrary $x = \sum_{n=1}^{\infty} c_n \psi_n \in X$ any t: $0 < t < 1$ and for any rearrangement σ satisfying (1) one has $x = \sum_{n=1}^{\infty} c_{\sigma(n)} h_{\sigma(n)}$.

We also describe all nondecreasing sequences $\omega = \{\omega_n\}$ such that the system $\{\omega_n h_n\}$ is a quasi-greedy basis in $L^1(0,1)$.

Some results on equivalence of Stromberg and Haar wavelets are obtained as well.

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2.26 Loukas Grafakos

Department of Mathematics, University of Missouri, UNITED STATES

Gabor Ridge Functions: theory and applications

We discuss a directionally sensitive time-frequency decomposition and representation of functions. The coefficients of this representation allow one to measure the "amount" of frequency the function (signal, image) contains in a certain time interval, and also in a certain direction. This has been previously achieved using a version of wavelets called ridgelets, developed by Candes, but in this work we discuss an approach based on time-frequency or Gabor elements. Applications to image processing and medical imaging are presented.

2.27 Philippe Jaming

MAPMO Université d'Orléans, FRANCE

Nazarov's uncertainty principle in higher dimension

In this talkwe prove that there exists a constant C such that, if S, Σ are subsets of $\mathbf{R}^d$ of finite measure, then for every function $f \in L^2(\mathbf{R}^d)$,

$$\int_{\mathbf{R}^d} |f(x)|^2 dx \leq Ce^{C\min(|S||\Sigma|,|S|^{1/d}w(\Sigma),w(S)|\Sigma|^{1/d})}\Big(\int_{\mathbf{R}^d\setminus S} |f(x)|^2 dx + \int_{\mathbf{R}^d\setminus\Sigma} |\hat{f}(x)|^2 dx\Big)$$

where $\hat{f}$ is the Fourier transform of f and $w(\Sigma)$ is the mean width of Σ. This extends to dimension $d \geq 1$ a result of Nazarov in dimension $d = 1$.

2.28 Norbert Kaiblinger

NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

Connecting discrete and continuous Gabor frames

We show how discrete-time Gabor frames can be used for approximating continuous-time Gabor frames.

2.29 Henning Kempka

Department of Mathematics, University of Jena, GERMANY

Generalized 2-microlocal Spaces

We give a generalization of 2-microlocal spaces in the sense of weighted Besov spaces. The concept of 2-microlocal analysis or 2-microlocal function spaces is due to J.M. Bony (see [1]). It is an appropriate instrument to describe the local regularity and the oscillatory behavior of functions near to singularities.

The approach is Fourier-analytical using Littlewood-Paley-analysis of distributions. The theory has been elaborated and widely used in fractal analysis and signal processing by several authors. We refer to [2], [3], [4], [5] and [6].

Therefore, let $\{\varphi_j\}_{j\in\mathbb{N}_0}$ be a smooth resolution of unity and let $\{w_j\}_{j\in\mathbb{N}_0}$ be a sequence of weight functions satisfying

$$0 < w_j(x) \le C w_j(y)(1 + 2^j|x-y|)^\alpha \quad (1)$$
$$2^{-\alpha_1} w_j(x) \le w_{j+1}(x) \le 2^{\alpha_2} w_j(x), \quad (2)$$

for $x, y \in \mathbb{R}^n$, $j \in \mathbb{N}_0$ and $\alpha, \alpha_1, \alpha_2 \ge 0$. Let $0 < p, q \le \infty$ and $s \in \mathbb{R}$. Then we introduce $B^{s,mloc}_{pq}(\mathbb{R}^n)$ as the space of all $f \in S'(\mathbb{R}^n)$ such that

$$\|f|B^{s,mloc}_{pq}(\mathbb{R}^n)\| = \left(\sum_{j=0}^{\infty} 2^{jsq} \|w_j \mathcal{F}^{-1}(\varphi_j \mathcal{F} f)|L_p(\mathbb{R}^n)\|^q \right)^{1/q} < \infty. \quad (3)$$

The usual 2-microlocal spaces $C^{s,s'}_{x_0}(\mathbb{R}^n)$, as described in [3], are a special case of (3) with $p = q = \infty$ and the weight functions

$$w_j(x) = (1 + 2^j|x - x_0|)^{s'} \quad \text{for some } x_0 \in \mathbb{R}^n. \quad (4)$$

We give first properties of these spaces and and a characterization in sequence spaces by wavelets. We follow closely the ideas expressed in [7].

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2.30 Ilya Krishtal

Department of Mathematical Sciences, Northern Illinois University, UNITED STATES

Slanted matrices, sampling, and Banach frames

2.31 Stefan Kunis

Faculty of Mathematics, Chemnitz University of Technology, GERMANY

Efficient reconstruction of functions on the sphere from scattered data

Recently, fast and reliable algorithms for the evaluation of spherical harmonic expansions have been developed. The corresponding sampling problem is the computation of Fourier coefficients of a function from sampled values at scattered nodes. We consider a least squares approximation and an interpolation of the given data. Our main result is that the rate of convergence of the two proposed iterative schemes depends only on the mesh norm and the separation distance of the nodes. In conjunction with the nonequispaced FFT on the sphere, the reconstruction of N^2 Fourier coefficients from M reasonably distributed samples is shown to take $\mathcal{O}(N^2 \log^2 N + M)$ floating point operations. Numerical results support our theoretical findings.

2.32 Richard S. Laugesen

Department of Mathematics, University of Illinois, UNITED STATES

Affine synthesis onto Lebesgue and Hardy spaces

Our goal is to synthesize surjectively onto the classical Lebesgue and Hardy spaces, from discrete analogues of those spaces, by using small-scale affine systems.

We assume very little about the synthesizing function except that it has nonvanishing integral. Thus the ability to decompose arbitrary functions into linear combinations of translates and dilates of the synthesizer requires no special properties of the synthesizer.

The corresponding analysis operator is shown to map the Lebesgue and Hardy spaces into discrete Lebesgue and Hardy spaces, respectively.

[Joint with H.-Q. Bui, U. of Canterbury]

2.33 Franz Luef

NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

The density theorem of Gabor analysis: an operator algebraic approach

We show how the density theorem of Gabor analysis follows from Rieffel's result on Morita equivalent pairs of von Neumann algebras.

2.34 Yurii Lyubarski

Department of Mathematical Sciences, Norwegian University of Science and Technology, NORWAY

Vector-valued Gabor frames with Hermite functions

2.35 Shobha Madan

Department of Mathematics, Indian Institute of Technology, Kanpur, INDIA

Structure of the spectrum and Fuglede's Conjecture for three intervals

We first prove that if $\Omega \subset \mathbb{R}$ is a union of n intervals and is a spectral set, with a spectrum contained in a lattice, then the spectrum is rational. The method of this proof has some implications on the *spectral implies tiling* part of Fuglede's conjecture for three intervals, which we prove under the hypothesis that the spectrum is contained in a lattice. At one step in the proof, we need a symbolic computation using *Mathematica* to reduce to the equal interval case. This latter case is then taken care of explicitly. Finally we also prove the converse part, *tiling implies spectral* for the three interval case.

2.36 W. R. Madych

Department of Mathematics, University of Connecticut, UNITED STATES

Harmonic Polysplines

Polysplines, christened and promoted by Ognyan Kounchev [1], are natural multivariate analogues of classical univariate piecewise polynomial splines. In this work we establish properties of harmonic polysplines that interpolate arbitrary distributions defined on parallel hyperplanes. In particular we show the following: (a) Such polysplines provide a solution to the natural minimization problem involving the class of distributions whose gradient is square integrable. (b) In the case when the data are polynomials such polysplines coincide with harmonic polynomials on the slabs which are determined by the parallel hyperplanes.

[1] O. Kounchev, *Multivariate Polysplines: Applications to Numerical and Wavelet Analysis*, Academic Press, San Diego, 2001.

2.37 Parasar Mohanty

Department of Mathematics, Indian Institute of Technology, Kanpur, INDIA

Extensions of Bilinear Multipliers

In this talk we will see Jodeit's type of transference of bilinear multipliers from circle group $\mathbb{T}$ to group of reals $\mathbb{R}$.

2.38 Sofia Olhede

Department of Mathematics, Imperial College London, UNITED KINGDOM

The Hyperanalytic Wavelet Transform

The extension of the analytic wavelet transform to higher dimensions is discussed. Such extensions require an appropriate definition of the extension of the analytic signal, and a careful combination of the calculation of an analytic signal with the operation of localization. The hyperanalytic wavelet transform is defined to ensure suitable properties of the transform coefficients.

2.39 Maria Perel

Department of Mathematical Physics, Physics Faculty, St. Petersburg University, RUSSIAN FEDERATION

New exact integral representations of solutions of the wave equation based on continuous wavelet analysis

Solutions of the wave equation with constant coefficients in a three-dimensional space are presented as superpositions of its localized solutions, which we call physical wavelets, following

G.Kaiser. The talk consists of two parts.

The first one is based on my works with M.S.Sidorenko [1-4]. The space of square integrable solutions is decomposed into a direct sum of two subspaces. In each subspace, the formalism of continuous wavelet analysis is developed. The choice of the mother physical wavelet is discussed. A family of wavelets is constructed from this wavelet with the help of transformations of dilation, spatial translation and rotation. Next the wavelet transform of an arbitrary solution from the subspace is defined. An isometry and a reconstruction formula are proved. The comparison of results with the results of G. Kaiser is given.

The second part contains an integral representation for solutions of the boundary-initial value problem for the wave equation in a half-space. Physical wavelets are constructed by dilation, translation and Lorentz transformations from four mother physical wavelets [5].

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[3] Perel M.V. and Sidorenko M.S., 2007, New physical wavelet 'Gaussian wave packet', Journal of Physics A: Mathematical and Theoretical 40, pp 3441-3461, http://stacks.iop.org/1751-8121/40/3441

[4] Perel M.V. and Sidorenko M.S., Wavelet-based integral representation for solutions of wave and Klein-Gordon equations, to appear

[5] Perel M.V. Wavelet-analysis in solving the boundary-initial value problem for the wave equation in a half-space, to appear

2.40 Dušanka Perišić

Department of Mathematics and Informatics, University of Novi Sad, SERBIA

Kernel Theorem for the space of generalized Gelfand-Shilov spaces

est space of tempered ultradistributions of Beurling - Komatsu and Roumieu Komatsu type can be identified (in both quasianalitic and nonquasianalitic case) with the spaces of sequences of ultrapolynomal falloff and their dual spaces with the space of sequences of ultrapolynomial growth. We prove these characterizations of the spaces and use them to give a simple proof of the kernel theorem for the space of tempered ultradistributions of Beurling - Komatsu type. As a consequence of the kernel theorem we have that the Weyl transform can be extended on a space of tempered ultradistributions of Beurling - Komatsu type.

2.41 Götz Pfander

Jacobs University Bremen, GERMANY

Sampling and sparsity: from vectors and functions to matrices and operators

We outline the identification problem for matrices and operators with ”sparse representations” and show how this leads to a generalization of Shannon’s sampling theorem and of the identifiability of time invariant operators to larger classes of pseudodifferential operators of any degree. In this overview talk we discuss operators with discretly supported spreading functions (Gabor frame operators), compactly supported spreading function (underspread operators) and bandlimited or compactly supported Weyl symbols.

We give applications to - wireless communication channels - multiple-input multiple-output channels - dequantization

2.42 Daniel Potts

Faculty of Mathematics, Chemnitz University of Technology, GERMANY

Fast Fourier transform at nonequispaced knots and applications

We use the recently developed fast Fourier transform at nonequispaced knots (NFFT) in a variety of applications. The NFFT realises the fast computation of the sums

$$f(w_j) = \sum_{k \in [-N/2, N/2)^d \cap \mathbb{Z}^d} \hat{f}_k \, \mathrm{e}^{-2\pi \mathrm{i} k w_j} \qquad (j = 0, \ldots, M-1)$$

where $w_j \in [-1/2, 1/2)^d$. Software: http://www.tu-chemnitz.de/∼potts/nfft

We discuss fast and reliable algorithm for the optimal interpolation of scattered data on the torus $\mathbb{T}^d$ by multivariate trigonometric polynomials as well as the approximation problem. The algorithm is based on a variant of the conjugate gradient method in combination with the fast Fourier transforms for nonequispaced nodes. We present a worst case analysis as well as results based on probabilistic arguments. The main result is that under mild assumptions the total complexity for solving the interpolation or approximation problem at M arbitrary nodes is of order $\mathcal{O}(N^d \log N + M)$. Finally, we apply these methods in magnetic resonance imaging. This talk based on joint results with S. Kunis (TU-Chemnitz), A. Böttcher (TU-Chemnitz) and H. Eggers (Philips Hamburg).

2.43 Alex Powell

Department of Mathematics, Vanderbilt University, UNITED STATES

Signal estimation from noisy dense measurements.

2.44 Abdumalik Rakhimov

National University of Uzbekistan, Department of Math Physics; Department of Mathematics, Federal Urdu University, Karachi, PAKISTAN

Localization of spectral expansions of distributions

1. Introduction. Classical analysis deals with the smooth or piecewise smooth functions. But many phenomena in nature require for its description either "bad" functions or even they can not be described by regular functions. Therefore, one has to deal with distributions that describe only integral characteristics of phenomena.

Application of the modern methods of mathematical physics in the spaces of distributions, leads to the convergence and sumability problems of spectral expansions of distributions. The convergence and sumability problems of spectral expansions of distributions, associated with partial differential operators, connected with the development of mathematical tools for modern physics.

Especially simple and important example is Fourier series of Dirac's delta function, partial sum of which is well known Dirichlet's kernel. From the classic theory of trigonometric series it is known that Dirichlet's kernel is not uniformly approximation of delta function. So spectral expansions of Dirac's delta function is not convergent in any compact set out of the support of the distribution. But arithmetic means of the partial sum of Fourier series of Dirac's delta function coincides with Fejer's kernel and in one dimensional case it uniformly convergent to zero in any compact set where delta function is equal to zero. In multidimensional case the problem become more complicated.

2. Spectral expansions connected with partial differential operators. Let Ω - an arbitrary N - dimensional domain. Consider a differential, elliptic, half bounded and symmetric operator $A(x,D) = \sum_{\alpha\leq 2m} a_\alpha(x)\cdot D^\alpha$ in Hilbert's space $L_2(\Omega)$ with domain of definition of $C_0^\infty(\Omega)$, here α is N - dimensional vector with non negative integer coordinates $\alpha = (\alpha_1, \alpha_2, ..., \alpha_N)$, $|\alpha| = \alpha_1+\alpha_2+....+\alpha_N$, $D_j = \frac{1}{i}\frac{\partial}{\partial x_j}$ and $D^\alpha = D^{\alpha_1}\cdot D^{\alpha_2}\cdot\dots\cdot D^{\alpha_N}$. Let $\hat{A}$ some selfadjoint extension of this operator in $L_2(\Omega)$ and $\{E_\lambda\}$ corresponding spectral family of projections. The projections $\{E_\lambda\}$ are integral operators with the kernels $\Theta(x,y,\lambda)$:

$$E_\lambda f(x) = \int_\Omega f(y)\Theta(x,y,\lambda)dy, \qquad f\in L_2(\Omega). \tag{5}$$

Function $\Theta(x,y,\lambda)$ is called spectral function of operator $\hat{A}$, and integral (1) is called spectral expansions of f corresponding to operator $\hat{A}$.

3. Problems of summability and localization of spectral expansion. One can study convergence and summability problems of spectral expansions of distributions in classical means in the domain where they coincide with regular functions. But singularities of the distribution still will be essential for convergence problems even at regular points as it was mentioned above in case of Delta function. For spectral expansions one can apply Riesz's method of summability or other regular methods (for instant Chezaro method). Reisz's means of order $s \geq 0$, of spectral expansion $E_\lambda f(x)$ define by equality:

$$E_\lambda^s f(x) = \int_\mu^\lambda \left(1 - \frac{t}{\lambda}\right)^s dE_t f(x). \tag{6}$$

We study summability problems for spectral expansions in different topologies and in different functional spaces. In particular we consider the problems of localization of spectral expansions. The problem of localization can be formulated in following way: *Let* f *an infinite differentialable function in neighborhood of a point* x_0. *What is the influence of smoothness (or non smoothness) of function* f *in some other points for the convergence of* $E_\lambda^s f(x)$ *in small neighborhood of the point* x_0.

4. Main results. Main results of the present work are obtaining sharp conditions for summability and localization of spectral expansions connected with elliptic partial differential operators in different spaces of distributions. It is obtained sharp relation between order of summation and smoothness of the distributions in Hilbert spaces. In case of Banach spaces sharp results established for multiple Fourier series and expansions connected with Laplace-Beltrami operator on sphere.

5. Some references. Below it is given a list of some papers devoted investigations of these problems.

see also: `http://univie.ac.at/nuhag-php/dateien/talks/541_Rakhimov's talk a`

2.45 Holger Rauhut

NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

Identification of matrices having a sparse representation

Motivated by the channel estimation problem in wireless communication matrices are considered that have a sparse representation in terms of elementary matrices. The basic problem under consideration consists of identifying such a matrix from its action on only one vector. This can be restated in terms of a sparse approximation or sparse recovery (compressed sensing) problem. So many algorithms from this field apply to perform the practical reconstruction, and there are direct consequences from known results on random measurement matrices, such as the Gaussian, Bernoulli and partial Fourier ensemble. However, the main focus will be on identifying an operator being the sum of a few time frequency-shifts (that is, it has sparse spreading function). Reconstruction results for basis pursuit (ℓ_1 minimization) will be presented. They can as well be interpreted as sparse approximation results for particular finite Gabor frames.

This is joint work with Gtz Pfander (International University of Bremen) and Jared Tanner (University of Utah).

2.46 Richard Rochberg

Department of Mathematics, Washington University St. Louis, UNITED STATES

Discrete Models for Spaces of Holomorphic Functions

2.47 Luigi Rodino

Department of Mathematics, University of Torino, ITALY

Fourier integral operators and multidimensional wavelets

2.48 Daniela Roşca

Department of Mathematics, Technical University of Cluj-Napoca, ROMANIA

Spherical quadrature formulas with equally-spaced nodes on latitudinal circles

In [2] we have constructed quadrature formulas on the 2-sphere, based on some fundamental systems of $(n+1)^2$ points ($n+1$ equally-spaced points taken on $n+1$ latitudinal circles), constructed by Laín-Fernández [1]. These quadratures are of interpolatory type, therefore the degree of exactness is at least n. In some particular cases, the degree of exactness can be $n+1$ and this exactness is the maximal one which can be obtained, based on the above mentioned fundamental system of points [3].

In this paper we try to improve the exactness by taking more equally-spaced points on each latitude and equal weights for each latitude. We show that the maximal degree of exactness which can be attained with $n+1$ latitudes is $2n+1$, and then we present some situations in which this exactness can be achieved.

Of a special relevance is the discussion of solvability of the system

$$\sum_{j=1}^{q} \alpha_j (e^{ix_j} + e^{iy_j}) = 0, \quad \sum_{j=1}^{q} \mu_j (e^{ix_j} - e^{iy_j}) = 0,$$

with n odd, $q = \frac{n+1}{2}$, $\alpha_j, \mu_j > 0$ satisfying the inequalities $\frac{\alpha_{j+1}}{\mu_{j+1}} \geq \frac{\alpha_j}{\mu_j}$ $(j = 1, \ldots, q-1)$ and $x_j, y_j \in [0, 2\pi)$ unknowns. We give some sufficient conditions for its solvability, respectively non-solvability, but we do not have yet a necessary and sufficient condition for its solvability.

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[2] J. Prestin and D. Roşca, *On some cubature formulas on the sphere*, J. Approx. Theory. 142 (2006), pp. 1–19.

[3] D. Roşca, *On the degree of exactness of some positive cubature formulas on the sphere*, Aut. Comp. Appl. Math., 15 (2006), no. 1, pp. 283–288.

2.49 Alexey Rozhdestvenskii

Department of General Control Problem, Moscow State University, RUSSIAN FEDERATION

On necessary conditions for Fourier multipliers

We plan to present conditions for a bounded function φ to be a Fourier multiplier of weak type $\big(\psi(L)(G), \psi(L)(G)\big)$, where $G = \mathbb{R}^d$ or $G = \mathbb{T}^d$, provided the Young function $\psi(t)$ grows slower than $t \ln^{1/2} t$ as t tends to infinity.

2.50 Ziemowit Rzeszotnik

Mathematical Institute, University of Wroclaw, POLAND

Banach algebras of pseudodifferential operators and their almost diagonalization

We define new symbol classes for pseudodifferential operators and investigate their calculus. The symbol classes are parametrized by commutative convolution algebras. To every solid convolution algebra over a lattice we associate a symbol class that is reminiscent of a modulation space. Then every operator with a symbol in such a class is almost diagonal with respect to special wave packets (coherent states or Gabor frames) and rate of almost diagonalization is described precisely by the underlying convolution algebra. Furthermore, the corresponding class of pseudodifferential operators is a Banach algebra of bounded operators on $L^2(R^d)$. If a version of Wiener's lemma holds for the underlying convolution algebra, then the algebra of pseudodifferential operators is closed under inversion. The theory contains as a special case the fundamental results about Sjöstrand's class and yields a new proof of a theorem of Beals. This is a joint work with Karlheinz Gröchenig.

2.51 Salti Samarah

Department of Mathematics, Jordan University of Science and Technology, JORDAN

Bernstein-type inequalities for modulation spaces

We give sufficient conditions for a tempered distribution to belong to certain modulation spaces by proving some Bernstein theorems for modulation spaces using nonlinear approximation method.

2.52 Karin Schnass

École Polytechnique Fédérale de Lausanne, SWITZERLAND

Average Performance Analysis for Thresholding

We show that with high probability the thresholding algorithm can recover signals that are sparse in a redundant dictionary as long as the *2-Babel function* is growing slowly. This implies that it can succeed for sparsity levels up to the order of the ambient dimension. The theoretical bounds are illustrated with numerical simulations. As an application of the theory *sensing dictionaries* for optimal average performance are characterised and their performance is tested numerically.

2.53 Thomas Strohmer

Department of Mathematics, University of California, Davis, UNITED STATES

The Unreasonable Effectiveness of Banach Algebras in Numerical Analysis

I will show that several problems arising at the interface of operator theory and numerical analysis can be solved in an elegant manner by using concepts from Banach algebra theory. In the first part of my talk I will consider the approximate solution of an infinite system of

linear equations via the classical finite section methods. Using Banach algebra methods we are able to derive quantitative convergence estimates for the finite section method. We also derive the first finite section method that can be applied to a large class of non-hermitian (and non-Toeplitz-type) matrices. In the second part of my talk I will investigate the interplay of matrix factorizations and Banach algebras. An important noncommutative generalization of the famous Wiener's Lemma states that under certain conditions the inverse A^{-1} of a matrix A will inherit the off-diagonal decay properties from A. I will show that this Wiener property also extends to certain matrix factorizations (such as QR or LU factorization) and matrix functions (such as the matrix exponential). I will discuss applications of the above results in signal processing and communications.

2.54 Jared Tanner

Department of Mathematics, University of Utah, UNITED STATES

The surprising structure of Gaussian point clouds and its implications for signal processing

We will explore connections between the structure of high-dimensional convex polytopes and information acquisition for compressible signals. A classical result in the field of convex polytopes is that if N points are distributed Gaussian iid at random in dimension n¡¡N, then only order $(logN)^n$ of the points are vertices of their convex hull. Recent results show that provided n grows slowly with N, then with high probability all of the points are vertices of its convex hull. More surprisingly, a rich ”neighborliness” structure emerges in the faces of the convex hull. One implication of this phenomenon is that an N-vector with k non-zeros can be recovered computationally efficiently from only n random projections with n=2e k log(N/n). Alternatively, the best k-term approximation of a signal in any basis can be recovered from 2e k log(N/n) non-adaptive measurements, which is within a log factor of the optimal rate achievable for adaptive sampling. Additional implications for randomized error correcting codes will be presented.

This work was joint with David L. Donoho.

2.55 Apostolos Vourdas

Department of Computing, University of Bradford, UNITED KINGDOM

Harmonic analysis on a Galois field

2.56 Ferenc Weisz

Department of Numerical Analysis, Eötvös University, HUNGARY

Some summability results for Gabor series

The a.e. convergence and the convergence in modulation and Wiener amalgam spaces are investigated. A general summability method, the so-called θ-summability is considered for Gabor series. Some classical theorems, that are well known for trigonometric Fourier series, are investigated for Gabor series.

2.57 Piotr Wojdyłło

Institute of Mathematics, Polish Academy of Sciences, POLAND

Non-Separable Wilson Tight Frames for 2D

We deal with an exemplary formula leading to the answer to H.G. Feichtinger question about existence of Wilson system for 2D /not being just a tensor of 2 classical Wilson systems/ and which is built from the elements of Gabor tight frame of redundancy 5.

2.58 Przemyslaw Wojtaszczyk

Instytut Matematyki Stosowanej, Uniwersytet Warszawski, POLAND

Tresholding and compressed sensing

3 Posters

3.1 Roza Aceska

Faculty for Mechanical Engineering, Skopje, MACEDONIA

Functions of Variable Bandwidth

In this presentation we construct a moderate weight in connection to a variable bandwidth strip in the time-frequency domain. The weight of positions within the strip is 1 and grows polynomially beyond the strip. We investigate the corresponding weighted function space, and show that it is a dilation-invariant modulation space. One can perform function reconstruction within the theory of coorbit spaces. The corresponding norm is - up to equivalence - independent of stretching or shrinking of the bandwidth.

3.2 Dominik Bayer

NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

Time-Frequency Localization Operators and the Berezin Transform

3.3 Frédéric Bernicot

Université Paris-Sud, FRANCE

L^p estimates for multi-linear operators, generalizing multilinear para-products

3.4 Ioan Goletz

University of Timisoara, ROMANIA

On almost periodic random signals

The signal prediction is one of major target that plays an important role in numerous applications. Time series show either a combination of periodic phenomena with stochastic components or chaotic behavior. Usually, the computing of nonlinear characteristics indicates the real complexity of the system. In many cases, the separation of frequency bands representing periodic or almost periodic behaviors, allows comprehension of the hidden nonlinear or stochastic phenomena involved. In this work a signal separation method of almost periodic components based on probabilistic norms is described. This method achieves effective time series forecasting needed in applications of fluctuations in time phenomena. At a given moment t the signal value of a random signal is assimilated to a random variable on a space with a probability measure. In this paper we adopt an another point of view, at each moment t the value of a random signal is known by its probability measure, that is, the value of a random signal at the moment t is considered an element of a probabilistic normed space. Let (Ω, K, P) be a complete probability measure space, i.e., the set Ω is a nonempty abstract set, $\mathcal{K}$ is a σ-algebra of subsets of Ω and P is a complete probability measure on $\mathcal{K}$. Let $(X, \mathcal{B})$ be a measurable space, where $(X, ||\cdot||)$ is a separable Banach space and $\mathcal{B}$ is the σ-algebra of the Borel subsets of $(X, ||\cdot||)$. A mapping f is said to be a random signal defined on the time subset A of real line with values in a separable Banach space X if, for each $t \in A$ the mapping $f(t) : \Omega \mapsto X$ is a X-valued random variable. But, the space of random variable can be endowed with a probabilistic normed space structure. So, we define a random signal as a mapping of a subset of a real line with values into a probabilistic normed space. This new framework is appropriate to give methods for approximation of random signals, for describing periodicity types, for defining the time variation at a moment$t$ and the informational energy on a time interval. [1]

see also: `http://univie.ac.at/nuhag-php/dateien/talks/771_stroblabstract.te`

3.5 Preben Graaberg Ness

Department of Mathematical Sciences, Norwegian University of Science and Technology, NORWAY

Fast space-scale filtering of the wavelet-transform of medical ultrasound images

The purpose of our investigation is to construct an efficient multi-scale edge-detector for medical ultrasound images using the continuous wavelet-transform. Typically such detectors require that the wavelet-transform is computed at several scales, merely in order to decide which information should be combined across scales. Our focus has been if/how one can determine which modulus-maxima are connected by a maxima-line if computing the wavelet-transform at only a few scales. We propose an algorithm which (when applied to medical ultrasound images) uses as few scales as possible.

The theoretical part of the investigation has focused on how the mutual amplitude and position of step-edges in a signal will influence the position and amplitude of the modulus-maxima across scales. This has been used to find bounds for which scales one has to compute the wavelet-transform, in order to ensure that one is able to determine which modulus-maxima are connected by a maxima-line.

Numerical investigation of the algorithm applied to rays in medical ultrasound images of brain-tumors, indicates that for such type of signals our algorithm is close to optimal if using scales $s = 2^j$, and still good when using scales $s = 2^{2j}$.

3.6 Niklas Grip

Department of Mathematics, Luleå University of Technology, SWEDEN

Deconvolution Based Analysis of Perturbed Integer Sampling in Shift-Invariant Spaces

An important cornerstone of both wavelet and sampling theory is *shift-invariant spaces*, that is, spaces spanned by a Riesz basis of integer-translates of a single function φ, which is referred to as *interpolating* if $\varphi(n) = \delta_{0,n}$ for integers n.

Under some mild differentiability and decay assumptions on $\widehat{\varphi}$, we show that φ is interpolating and generates a shift-invariant space V if and only if $\widehat{\varphi}(\xi) = \int_{\xi-\pi}^{\xi+\pi} g(\nu)d\nu$ for a certain function g with $\int_{\mathbf{R}} g(\xi)d\xi = 1$.

[1]AMS (1990) Subject Classification : 54E35, 46A19.
Key words and phrases: probabilistic normed space, random signal

Further, we exploit this fact in combination with analysis techniques introduced in a previous paper to derive jitter bounds $\varepsilon = \sup_{k\in\mathbb{Z}} |\varepsilon_k|$ for which any $f \in V$ can be reconstructed from perturbed integer samples $f(k+\varepsilon_k)$.

Finally, we demonstrate the resulting sampling theorem, for example, for some Meyer-type φ and for compactly supported positive g with bounded variation.

The presented results are joint work with Stefan Ericsson.

3.7 Anna Gryboś

Faculty of Applied Mathematics, UST-AGH, Krakow, POLAND; NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

Weighted Gabor Frames

Regular Gabor families with the semi-normalized weights put on atoms are considered and the effect of the weights on the dual Gabor families is studied. Approximation of the obtained dual family by the original dual family with the appropriately chosen weight applied is analysed. Several examples are presented for illustration.

3.8 Sigrid Bettina Heineken

Department of Mathematics, University of Buenos Aires, AUSTRIA

Sampling Spaces

We consider the sampling problem in shift invariant spaces of $L^2(\mathbf{R})$, generated by functions which integer translates are a frame for the space. In particular we study sampling spaces. We characterize the functions that belong to sampling spaces and we obtain atomic decompositions of these spaces in sampling subspaces.

3.9 Sunggeum Hong

Department of Mathematics, Chosun University, KOREA, REPUBLIC OF

Weak type estimates for cone type mutipliers associated with convex polygons

Let $\mathcal{P}$ be a convex polygon in $\mathbb{R}^2$ which contains the origin in its interior. Let ρ be the associated Minkowski functional defined by $\rho(\xi) = \inf\{\epsilon > 0 : \epsilon^{-1}\xi \in \mathcal{P}\},\ \ \xi \neq 0.$ We consider the family of convolution operators T^δ associated with cone type multipliers

$$\big(1 - \frac{\rho(\xi)^2}{\tau^2}\big)_+^\delta, \quad (\xi,\tau) \in \mathbb{R}^2 \times \mathbb{R},$$

and show that T^δ is of weak type (p,p) on $H^p(\mathbb{R}^3)$, $1/2 < p < 1$ for the critical value $\delta = 2(1/p - 1)$.

This is a joint work of Sunggeum Hong, Joonil Kim and Chan Woo Yang.

3.10 Hamidreza Karimi

Faculty of Mathematics and Computer Science, University of Bremen, GERMANY

Multisensor Image Fusion Using Redundant Wavelet Transform

The standard data fusion methods may not be satisfactory to merge multisensor images. Wavelets as powerful signal processing tools in dealing with nonstationary signals provide an alternative for data fusion. In this paper a methodology for multisensor image fusion using redundant wavelet transform is proposed. The aim of our work is to use redundant wavelet transform for a more effective extraction of dominant features as observed in different scales. We have used wavelet transform and pyramid structure of wavelet analysis for signal representation in time-scale domain and to compute dominant signal components at different time/scale resolutions. Redundancy of the wavelet transform is used to enhance extraction and fusion of all significant features coming from multisensor images. A fusion rule will be developed that allows taking into account the dominant features of multisensor images at different scales. Simulations will be used to show the effectiveness of the proposed methodology.

3.11 Damiana Lazzaro

Department of mathematics, University of Bologna, ITALY

An Adaptive Thresholding Algorithm for 3D Images Denoising

Reducing noise in images is a preliminary step in many image processing applications. In this work a method for edge-preserving denoising of three dimensional image data is presented. It is based on the 3D generalization of the dyadic wavelet transform and overcomes the problems presented by the extension to higher dimensions of the classical orthogonal wavelet transforms. In fact, a straightforward extension of these separable transforms to multidimensional data gives very poor results, due to the lack of translation invariance that produces undesired artifacts. On the contrary, dyadic wavelets are truly multidimensional translation invariant bases, and the coefficients of the dyadic expansion of an image f at different scales contain informations of the gradient of f smoothed according to the scale. By integrating these geometric informations in the smoothing process we obtain a 3D denoising algorithm capable to preserve as much as possible of the signal features while reducing the noise to a sufficiently low level.

3.12 Jakob Lemvig

Department of Mathematics, Technical University of Denmark, DENMARK

The Canonical and Alternate Duals of a Wavelet Frame

We show that there exists a frame wavelet ψ with fast decay in the time domain and compact support in the frequency domain generating a wavelet system whose canonical dual frame cannot be generated by an arbitrary number of generators. On the other hand, there exists infinitely many alternate duals of ψ generated by a single function. Our argument closes a gap in the original proof of this fact by Daubechies and Han [Appl. Comp. Harmonic Anal. **12** (2002), no. 3, 269–285].

It is a well-known fact that every orthonormal wavelet $\phi \in L^2(R)$ with $|\hat{\phi}|$ continuous and $\hat{\phi}(\xi) = \mathcal{O}(|\xi|^{-1/2-\delta})$ as $|\xi| \to \infty$ for some $\delta > 0$ is associated with an MRA. The ψ constructed in this article is an example of a non-GMRA C^∞ frame wavelet with rapid decay.

In [Acta Appl. Math. **89** (2005), no. 1-3, 251–270] Baggett et al. gave an example of a non-MRA C^r tight frame wavelet with rapid decay.

[joint with Marcin Bownik, U. of Oregon].

3.13 Damián Marelli

NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

A Continuous-Time System Identification Method for Slowly Sampled Data

Both direct and indirect methods exist for continuous-time system identification. A direct method estimates continuous-time input and output signals from their samples and then use them to obtain a continuous-time model, whereas an indirect method estimates a discrete-time model first. Both methods rely on fast sampling to ensure good accuracy. In this presentation, we propose a more direct method where a continuous-time model is directly fitted to the available samples. This method produces an exact model asymptotically, modulo some aliasing ambiguity, even when the sampling rate is relatively low.

3.14 Azita Mayeli

Center for Mathematics, Munich University of Technology, GERMANY

Nearly Tight Wavelet Frames for Stratified Lie Groups

3.15 Markus Neuhauser

NuHAG, Faculty of Mathematics, University of Vienna, AUSTRIA

The metaplectic representation for finite cyclic groups

The purpose of this poster is to construct the metaplectic representation of the special linear group of a cyclic group with an odd number of elements as a projective representation steming from intertwining operators of the simplest Heisenberg group whose entries are elements of a finite cyclic group. This construction is done in an elementary way as well as the determination of the corresponding 2-coboundary to obtain an ordinary representation from the projective representation.

3.16 Sofian Obeidat

Department of Mathematics, Jordan University of Science and Technology, JORDAN

On a property of frames in A Hilbert space

We study some properties of frames in a separable Hilbert space which fall in the area of constructing new frames from old ones. Our work is motivated by the work done by Holub in 2004 on bases.

3.17 Serena Papi

Department of mathematics, University of Bologna, ITALY

Feature Preserving Image Denoising using Inter-and-intrascale Dependencies of Wavelet Coefficients

A new wavelet shrinkage algorithm for edge preserving image denoising is proposed. It represents an improvement of a previously presented thresholding method [1] that adds to the denoising process the construction of the map of image edges. The map is obtained by taking into consideration both space-scale dependencies and interscale correlations between corresponding coefficients of the dyadic Mallat's wavelet decomposition of the image. The knowledge of the location and the orientation of the edges allows us to perform an anisotropic shrinkage by killing the gradient components along the edges while keeping or only slightly shrinking the orthogonal components. The non-edge coefficients are then shrinked using a correlated Perona-Malik rule.
The resulting anisotropic wavelet thresholding scheme succeeds in producing a denoised version of the original image that still presents well-defined edges, while the noise components are removed.

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3.18 Hammad Qureshi

Department of Computer Science University of Warwick, UNITED KINGDOM

Comparative Analysis of Discriminant Wavelet Packet Features and

The idea of multiresolution analysis has been around for over two decades now. In this paper, we explore a multiresolution analysis based technique for histopathological image classification and compare it with raw image analysis. The principle idea for the former is to derive an optimal wavelet representation, called Adaptive Discriminant Wavelet Packet Transform (ADWPT), of the images in order to achieve the largest discrimination power. Our results show that the textural features combined with ADWPT yield a significant improvement in classification accuracy.

3.19 Pavel Rajmic

Brno University of Technology, CZECH REPUBLIC

The algorithm of Segmented wavelet transform

The method of segmented wavelet transform (SegWT) allows computing of the discrete-time wavelet transform of a signal segment by segment. This means that the method could be utilized for wavelet type processing of a signal in "real time", or in case we need to process a long signal (not necessarily in real time), but there is insufficient computational memory capacity for it (for

example in the DSPs). Then it is possible to process the signal part by part with low memory costs by this method.

3.20 Peter Rashkov

Jacobs University Bremen, GERMANY

Uncertainty in time-frequency representations on finite abelian groups

Classical and recent results on the uncertainty rinciple for functions on finite abelian groups relate the cardinaity of the support of a function to the cardinality of the support of its Fourier transform. We use these results and their proofs to obtain similar results relating the support sizes of functions and their short-time Fourier transforms.

3.21 Sandra Saliani

Dipartimento di Matematica, Universit della Basilicata, ITALY

On Stable Refinable Function Vectors with Arbitrary Support

Refinable function vectors with arbitrary support are considered. In particular necessary conditions for stability are given and a characterization of the symbol associated with a stable refinable function vector in terms of the transfer operator is provided: this is a generalization of Gundy's theorem to the vector case. The proof adapts the tools provided in Saliani, On stability and orthogonality of refinable functions, Appl. Comp. Harm. Anal., 21, (2006), 254-261. Though complications arise from noncommuting matrix products, the fundamental ideas are the same.

3.22 Peter Søndergaard

Department of Mathematics, Technical University of Denmark, DENMARK

The Linear Time Frequency Analysis Toolbox, version 1.0

The Linear Time Frequency Analysis Toolbox is a Matlab/Octave/C toolbox for doing time-frequency analysis. It is intended both as an educational and computational tool. The toolbox provides the basic Gabor, Wilson and MDCT transform along with routines for constructing windows (filter prototypes) and routines for manipulating coefficients. In addition to the pure transforms the toolbox supplies two kinds of time-frequency operators: Gabor multipliers and spreading operators. The usage of these tools are demonstrated by examples included in the toolbox.

3.23 Diana Stoeva

University of Architecture, Civil Engineering and Geodesy, BULGARIA

Frames for Frechet spaces

The motivation behind the present work comes from Banach frames and X_d-frames. During the last years, series expansions in a Banach space X and its dual X^* via a Banach frame or an X_d-frame have been of interest.

In the present talk we extend the concept of a frame to Fréchet spaces. We consider family of Banach spaces and determine conditions, implying series expansions in their projective and inductive limit. The results are applicable to some important Fréchet spaces, for example the Schwartz class S of rapidly decreasing functions, which can be written as projective limit of certain modulation spaces.

The present work is joint with S. Pilipovic and N. Teofanov.

3.24 Maja Tasković

Department of Mathematics, University of Novi Sad, SERBIA AND MONTENEGRO

A structural theorem for generalized Gelfand-Shilov spaces, quasi-analytic and nonquasianalytic case

Generalized Gelfand-Shilov spaces S^{Mp} are subspaces of Denjoy-Carleman classes C^{Mp} which are appropriate for Fourier Analysis. Examples of generalized Gelfand-Shilov spaces are Sato space F, Braun-Meise-Taylor space and all non-trivial Gelfand-Shilov spaces.

We prove that every element of the dual space $S^{Mp\prime}$ can be represented as the action of a partial-differential operator of infinite order on a continuous bounded function.

3.25 Zlatko Udovićić

Department of Mathematics, Faculty of Sciences, University of Sarajevo, BOSNIA AND HERZEGOVINA

Determining the Threshold of Compression in the Wavelet Transform with Orthonormal Wavelets

We described one nonstandard way for determining the threshold of compression in dependance of the allowed relative error of reconstruction. By using the same technique we determined the optimal threshold of compression. The basic characteristic of the presented idea is that we used geometrical interpretation of the pyramidal algorithm together with the basic definitions of the theory of probability.

3.26 Vitezslav Vesely

Department of Applied Mathematics and Computer Science, Masaryk University, CZECH REPUBLIC

Four-step Basis Pursuit with Applications

When solving real problems there is often missing a reliable theory behind them. In such situations the ideas about a choice of an appropriate model are very vague and produce models where it is hard to balance the requirement on sufficient regularity of the model (as few parameters as possible to guarantee numerical stability) and feasible precision which forces the analyst to increase the number of model components typically leading to overparametrization accompanied with non-uniqueness and numerical instability of solutions.

The standard estimation algorithms use to fail due to numerical instability caused by strong overparametrization. In [3] there was implemented a computationally intensive sparse parameter estimation technique based on BPA4 — a four-step modification of the Basis Pursuit Algorithm originally suggested by Chen et al [1] for time-scale analysis of digital signals and utilizing numerical procedure [2].

[3] is a collection of functions allowing one to construct and manipulate big finite frames in any abstract Hilbert space $\mathcal{H}$ with arbitrarily parametrized user-defined frame atoms going far beyond the common shift/scale/modulation schemes widely used for spectral representation of signals. Then BPA4 serves as a universal tool both for finding a stable sparse frame expansion approximating any object from or outside of $\mathcal{H}$ and possibly establishing the appropriate dual frame atoms if necessary.

In addition to some minimal theoretical background this contribution demonstrates performance and flexibility of BPA4 on four problems coming from completely diverse application fields: kernel approximation and smoothing (denoising) [4], improved time series forecasting within an overcomplete stochastic frame of type ARMA [5], analysis of air pollution by suspended particulate matter [6] and ROC curve estimation [7]. This new computationally intensive approach allowed us to reliably identify nearly zero parameters in the respective model and thus to find numerically stable sparse solutions.

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3.27 Patrik Wahlberg

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Weyl product algebras and modulation spaces

We discuss algebraic properties of the Weyl product acting on modulation spaces. For a certain class of weight functions w we prove that $M_w^{p,q}$ is an algebra under the Weyl product if $p \in [1, \infty]$ and $1 \leq q \leq \min(p, p')$. For the remaining cases $p \in [1, \infty]$ and $\min(p, p') < q \leq \infty$ we show that the unweighted spaces $M^{p,q}$ are not algebras under the Weyl product. The work has been done jointly with A. Holst and J. Toft.

3.28 Anastasia Zakharova

Department of Mechanics, Moscow State University, RUSSIAN FEDERATION

On the properties of generalized frames

The author introduces the notion of the generalized frame and considers its properties. Discrete and integral frames represent particular cases of generalized frames. Necessary and sufficient conditions for a generalized frame to be an integral (discrete) one are obtained. It is also proved that for any bounded invertible operator A from Hilbert space H (also from non-separable one) to $L_2(\Omega)$ (where Ω is a space with countably additive measure) with inverse bounded, there exists such a generalized system that A translates any element $x \in H$ to its expansion coefficient.

Key words: frame, dual frame, generalized system.

3.29 Georg Zimmermann

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Polynomial Reproduction: The unstable case

The question of polynomial reproduction is important in subdivision, since it is closely related to approximation order. It has been studied in the stable case by Cabrelli, Heil and Molter as well as Jetter and Zimmermann. We present results for the unstable case.